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Measurement bootstrapping Kalman filter

Zhentao Hu, Yumei Hu*, Yong Jin, Shanshan Zheng

Institute of Image Processing and Pattern Recognition, Henan University, No. 1, Jinming Road, Kaifeng 475004, Henan Province, People's Republic of China

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ABSTRACT

In practice, many estimation problems rely on on-line solutions, hence recusive methods are necessary. Tradition estimation approaches for linear systems subject to Gaussian noise are based on the Kalman filter and its improving algorithms. Based on the realization construction of state prediction and measurement update, Kalman filter can obtain the optimal estimation of state under the linear minimum variance criterion. However, it is known that the optimal filtering result is achieved in statistical sense, which can not meet a superior result for single filtering process because of the random characteristics effect of measurement noise. Aiming at this problem, the authors propose a novel Kalman filtering algorithm based on measurement bootstrapping strategy. In realization of algorithm, first, combining with the extraction and utilization of measurement information including the latest measurement and the prior statistical information from measurement noise modeling, the measurement bootstrapping strategy is designed and virtual measurements are sampled. Its objective is to enhance the reliability of latest measurement by increasing samples numbers. Second, virtual measurements are applied to Kalman filtering framework, and the key improvement is concentrated in the utilization of measurement information. In addition, considering the requirements in engineering application such as instantaneity, filtering precision and robustness, the distributed weighted fusion structure and the centralized consistency fusion structure are designed respectively. Finally, the theoretical analysis and experimental results verify the feasibility and efficiency of the proposed algorithm.

1. Introduction

System estimation problems are always encountered in the area of image processing, communication, fault diagnosis, automatic control, etc. The so-called estimation is to approximate real system state as exact as possible, through extracting and utilizing useful information from measurement with random disturbance. According to whether the property of estimated object changes over time, estimation problems are normally divided into two categories: parameter estimation and state estimation [1]. In fact, the interior relation exists between parameter estimation and state estimation. State estimation can be obtained by combining parameter estimation methods and internal regularity of dynamic random process or sequences. Some classical parameter estimation methods include least squares estimation, maximum likelihood estimation, maximum posteriori estimation, linear minimum variance estimation [2], etc. Based on Bayesian estimation principle, Kalman filter (KF) gives a typical implementation in the linear minimum variance criteria. KF belongs to a linear unbiased recursive filter. Linear denotes the filtering output is a linear function of measurement, unbiasedness denotes the real state and estimation state have the same mean, and recursive property requires current estimation can be obtained through the correction of prior estimation using latest measurement [3]. However, the prerequisite of the minimum variance is that the estimated system needs to meet three strictly assumptions including linear, Gaussian noise and noise independence. These conditions limit its application in nonlinear and colored noise systems.

In recent years, theoretical researches in KF focus on two aspects, one is to relax the application conditions. Namely, how to use the algorithm framework of KF better for state nonlinear and noise related problems. On one hand, aiming to the state estimation of nonlinear system, combined with local linearization, UT transform, Stirling interpolation technique, third-degree spherical-radial rule and stochastic sampling method, some experts and scholars have proposed the extended Kalman filter (EKF) [4], unscented Kalman filter (UKF) [5,6], central difference Kalman filter (CDKF) [7], cubature Kalman filter (CKF) [8,9], particle filter (PF) [10,11], ensemble Kalman filter (EnKF) [12,13] and a series of relevant improved algorithms, and filtering performance are obtained for specific object and background. It is worth mentioning that the data assimilation in EnKF provide a new approach in improving

* Corresponding author. Tel.: +86 15226009563.
E-mail address: hym_henu@163.com (Y. Hu).

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the adverse effect on filtering precision caused by single measurement uncertainty. And the design of measurement bootstrapping strategy in paper is also motivated by this method. On the other hand, aiming to the noise related problem, state dimension augment, measurement dimension augment and model reconstruction are most commonly treatment methods. The other aspect is about how to improve Kalman filtering precision. It is known that filtering precision of KF is restricted by three main arguments. First, the selection of filtering initial value, according to the implementation mechanism of KF, the initial value is unrelated to steady-state error, but it has an effect on the convergence time. Second, the error modeling of system noise, this parameter depends on the cognition of system state evolution. Third, the error modeling of measurement noise, it depends on sensor's accuracy. The higher accuracy of sensor leads to the better filtering performance. Aiming at the third argument, the simplest solution is to select sensor with higher accuracy. However, it is no doubt that the hardware cost will be increased correspondingly. In addition, a common solution is to utilize multi-source information fusion technology [2]. Namely, it takes a mechanism of combining software and hardware. In order to obtain a satisfactory filtering precision, the number of sensors needs to be configured rationally and weighted fusion methods need to be exactly selected. As a result, this method is lack of universality to objects in application. To solve the above problem, a novel the realization framework of KF based on measurement bootstrapping strategy is proposed, and the filtering precision is improved without additional hardware cost. The algorithm realization depends on the further extraction and utilization of the priori modeling information of measurement system. In addition, implementation structure is given in two forms. Namely, distributed weighted fusion and centralized consistency weighted fusion, and the algorithm performances are compared and analyzed in simulation experiments.

2. The standard Kalman filter

Consider a linear-Gaussian time-varying system state space model

$$\boldsymbol{x}_k = \boldsymbol{F}_{k-1}\boldsymbol{x}_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{u}_{k-1} \tag{1}$$

$$\boldsymbol{z}_k = \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k \tag{2}$$

where $\boldsymbol{x}_k$ and $\boldsymbol{z}_k$ are the state and the measurement vector at time k, respectively. $\boldsymbol{F}_{k-1}$ and $\boldsymbol{H}_k$ are known as matrices with compatible dimension, The process noise $\boldsymbol{u}_{k-1}$ and the measurement noise $\boldsymbol{v}_k$ are uncorrelated zero-mean white Gaussian processes, and $\boldsymbol{u}_k \sim \boldsymbol{N}\left(0, \boldsymbol{\sigma}^2_{\boldsymbol{u}_k}\right)$, $\boldsymbol{v}_k \sim \boldsymbol{N}\left(0, \boldsymbol{\sigma}^2_{\boldsymbol{v}_k}\right)$. The goal of state estimation is to approximate real state based on measurement sequences $\boldsymbol{Z}_k = \left\{\boldsymbol{z}_1, \boldsymbol{z}_2, \ldots, \boldsymbol{z}_k\right\}$. Based on Bayesian estimation principle, KF is recursive estimation in linear minimum variance criteria. And it contains two processes: state one-step prediction and measurement update. The specific implementation is as follows.

First, initializing state $\hat{\boldsymbol{x}}_{0|0}$ and estimation error covariance $\boldsymbol{P}_{0|0}$

$$\hat{\boldsymbol{x}}_{0|0} = \boldsymbol{x}_0 \tag{3}$$

$$\boldsymbol{P}_{0|0} = \boldsymbol{P}_0 \tag{4}$$

Second, calculating state one-step prediction $\boldsymbol{z}^i_{k,m}$ and state one-step prediction error covariance $\boldsymbol{P}_{k|k-1}$ based on the priori information of system model

$$\hat{\boldsymbol{x}}_{k|k-1} = \boldsymbol{F}_{k-1}\hat{\boldsymbol{x}}_{k-1|k-1} \tag{5}$$

$$\boldsymbol{P}_{k|k-1} = \boldsymbol{F}_{k-1}P_{k-1|k-1}\boldsymbol{F}^T_{k-1} + \boldsymbol{\Gamma}_{k-1}\boldsymbol{\sigma}^2_{u_{k-1}}\boldsymbol{\Gamma}^T_{k-1} \tag{6}$$

Finally, calculating current state estimation $\hat{\boldsymbol{x}}_{k|k}$ and its error covariance matrix $\boldsymbol{P}_{k|k}$

$$\boldsymbol{K}_k = \boldsymbol{P}_{k|k-1}\boldsymbol{H}^T_k\left(\boldsymbol{H}_k\boldsymbol{P}_{k|k-1}\boldsymbol{H}^T_k + \boldsymbol{\sigma}^2_{v_k}\right) \tag{7}$$

$$\hat{\boldsymbol{x}}_{k|k} = \hat{\boldsymbol{x}}_{k|k-1} + \boldsymbol{K}_k\left(\boldsymbol{z}_k - \boldsymbol{H}_k\hat{\boldsymbol{x}}_{k|k-1}\right) \tag{8}$$

$$\boldsymbol{P}_{k|k} = \boldsymbol{P}_{k|k-1} + \boldsymbol{K}_k\boldsymbol{H}_k\boldsymbol{P}_{k|k-1} \tag{9}$$

Note that $\boldsymbol{K}_k$ is the filter gain matrix at time k, it is used to measure the utilization level of the latest measurement in current state estimation.

3. The Kalman filter based on measurement bootstrapping strategy

The standard KF is able to obtain linear unbiased minimum variance estimation, when the system is linear and Gaussian. In algorithm mechanism, the state one-step prediction link is mainly used to achieve the utilization of the modeling information of system state evolution, while the measurement update link is mainly used to achieve the extraction of the latest measurement. Therefore, to improve the cognitive of estimated system and improving modeling accuracy of system state evolution process is a valid way to enhance the state estimation precision. However, the method restricted by objective conditions in some practical engineering application is difficult to achieve. For example, the estimated system state is actually unknown and/or uncertain. In addition, to rich measurement information and combine with multi-source information fusion technology is another possible way to enhance state estimation precision. But if multiple sensors are adopted, the hardware cost of measurement system will increase inevitably. Meanwhile, the choice of sensors accuracy, position configuration and random fault of sensors need be taken into account. Based on the above analysis, the advantages of multi-source information fusion method for data processing can be used for reference. Through building and utilizing virtual measurements rationally, the measurement reliability of system state is enhanced without increasing hardware cost (the number of sensors and accuracy), so that the adverse effect on filtering precision caused by measurement noise randomness is degenerated. On this basis, the distributed weighted fusion structure and the centralized consistency fusion structure are two implementations designed respectively.

3.1. The measurement bootstrapping strategy

At current time k, the measurement $\boldsymbol{z}_k$ of estimated system, there is measurement noise statistics information, namely, measurement noise variance $\boldsymbol{\sigma}^2_{\boldsymbol{v}_k}$. Besides, $\boldsymbol{H}_k\boldsymbol{x}_k$ related to modeling. The value of $\boldsymbol{\sigma}^2_{\boldsymbol{v}_k}$ depends on sensor's accuracy. Considering the two types of information mentioned above, the virtual measurement are built as follows

$$\begin{aligned} \boldsymbol{z}^i_{k,m} &= \boldsymbol{z}_k + \sum_{m=1}^{M}\boldsymbol{v}^i_{k,m} \\ &= \boldsymbol{H}_k\boldsymbol{x}_k + \boldsymbol{v}_k + \sum_{m=1}^{M}\boldsymbol{v}^i_{k,m} \end{aligned} \qquad m = 1, 2, \ldots, M;\ i = 1, 2, \ldots, N \tag{10}$$

where, $\boldsymbol{z}^i_{k,m}$ represents the ith virtual measurement in virtual measurement set. $\boldsymbol{v}^i_{k,m}$ and $\boldsymbol{v}_k$ are zero mean Gaussian noise, and $\mathrm{Cov}\left[\boldsymbol{v}^i_{k,m}, \boldsymbol{v}_k\right] = 0$, $\mathrm{Cov}\left[\boldsymbol{v}^i_{k,m}, \boldsymbol{v}^\zeta_{k,m}\right] = \begin{cases} \boldsymbol{\sigma}^2_{\boldsymbol{v}_k} & i = \zeta \\ 0 & i \neq \zeta \end{cases}$. The real measurement $\boldsymbol{z}_k$ is a constant when it is obtained from the sensor system at time k. Therefore $\boldsymbol{z}^i_{k,m}$ and $\boldsymbol{z}^j_{k,m}$ are