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Analysis on polarization characteristics of electromagnetic propagation in anisotropic magnetized plasma media

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ABSTRACT

The electromagnetic (EM) wave propagation through an anisotropic magnetized plasma layer is studied using the finite-difference time-domain (FDTD) method based on the Runge–Kutta exponential time differencing (RKETD) technique. When the propagation direction of the EM wave is perpendicular to the external magnetic direction, Voigt effect should be considered. The RKETD–FDTD formulations are derived in detail and are confirmed by computing the reflection and transmission coefficients for the ordinary polarized wave and extraordinary polarized wave through a magnetized plasma slab. Excellent agreement between the numerical results and the exact analytical solutions is demonstrated. The EM wave become partly longitudinal and partly transverse in magnetized plasma medium due to the Voigt effect are also proved.

1. Introduction

The finite-difference time-domain (FDTD) method is a widely used numerical technology to analyze electromagnetic (EM) wave radiation, propagation and scattering problems [1,2]. It has been used in a wide range of applications including modeling and design of microwave structures, pulse propagation in dispersive media and many other applications. During the past decade, the FDTD method has been used to simulate the transient solutions of electromagnetic wave propagation in various dispersive media. Plasma is a frequency-dependent dispersive medium, which can be modeled using the FDTD method. The frequency dependent FDTD formulation was first proposed by Luebbers et al., and the method is therefore known as the recursive convolution (RC) FDTD method [3,4]. Other algorithms include the auxiliary differential equation (ADE) FDTD method [5], the Z transform (ZT) FDTD method [6]. Many efforts have been devoted to improve the efficiency and accuracy of the above methods. These algorithms include the piecewise linear recursive convolution (PLRC) method [7], trapezoidal recursive convolution (TRC) FDTD method [8,9], the current density convolution (JEC) FDTD method [10], piecewise linear current density recursive convolution (PLCDRC) FDTD method [11], exponential time differencing (ETD) FDTD method [12], Runge–Kutta exponential time differencing (RKETD) method [13] etc. All of the above FDTD methods for magnetized plasma media require that the external magnetic field direction must be parallel to the direction of propagation, which is a serious limitation. For many practical cases of interest, however, the angle between the external magnetic field direction and the direction of propagation is perpendicular. The electric field vector of the incident wave can be polarized either parallel or perpendicular to the magnetic axis. The wave with its electric field vector parallel to the magnetic field is called the ordinary wave. The wave with its electric field vector perpendicular to the magnetic axis is called the extraordinary wave [14].

In this paper, the RKETD–FDTD is extended to analyze the polarization characteristics of EM wave propagation in magnetized plasma with perpendicular magnetic declination. When the propagation direction of the EM wave is perpendicular to the external magnetic direction, Voigt effect should be considered. The equations for magnetized plasma are derived in detail and are confirmed by computing the reflection and transmission coefficients for the ordinary polarized wave and the extraordinary polarized wave through a magnetized plasma layer. Excellent agreement between the numerical results and the exact analytical solutions is presented. The EM wave become partly longitudinal and partly transverse in magnetized plasma due to the Voigt effect are also proved.

2. Computational theory and RKETD–FDTD formulation

The time-dependent Maxwell's curl equations and constitutive relation in magnetized plasma are given by

$$\nabla \times \mathbf{H} = \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} + \mathbf{J}, \quad (1)$$

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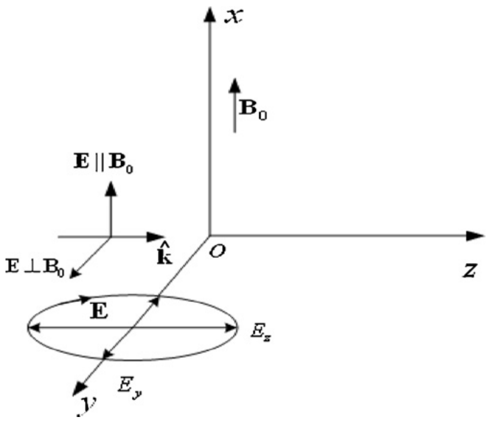

Fig. 1. Coordinate system used in analysis of EM waves that propagate in magnetized plasma.

$$\nabla \times \mathbf{E} = -\mu_0 \frac{\partial \mathrm{H}}{\partial t}, \tag{2}$$

$$\frac{d\mathbf{J}}{dt} + \nu \mathbf{J} = \varepsilon_0 \omega_p^2 \mathbf{E} + \boldsymbol{\omega}_b \times \mathbf{J}, \tag{3}$$

where **E** is the electric field, **H** is the magnetic intensity, **J** the polarization current density, ε_0 the permittivity of free space, μ_0 the permeability of free space, ν the electron collision frequency, ω_p^2 is the square of plasma angular frequency, $\boldsymbol{\omega}_b = e\mathbf{B}_0/m$ the electron gyrofrequency, $\mathbf{B}_0$ the external static magnetic field, and e and m are the electric charge and mass of an electron, respectively.

Assume that the propagation direction of the EM waves is perpendicular to the external biasing field. To simplify the algebra, a Cartesian coordinate system where the x-axis lies along the external static magnetic field $\mathbf{B}_0$ and where the z-axis lies along vector **k** (Fig. 1) is chosen. There are two choices: **E** can be parallel to $\mathbf{B}_0$ or perpendicular to $\mathbf{B}_0$.

If **E** is parallel to $\mathbf{B}_0$, the wave in this case is unaffected by the introduction of the magnetic field $\mathbf{B}_0$. As with the longitudinal plasma oscillations for parallel propagation, this is because the electric field E_x makes the particles move parallel to $\mathbf{B}_0$ and therefore produces no Lorentz force. The results are the same as EM wave in unmagnetized media. Eq. (1) is discretized using the Yee grid and leap-frog integration [1]

$$E_x^{n+1} = E_x^n - \frac{\Delta t}{\varepsilon_0 \Delta z}(\mathbf{H}_y^{n+1/2} - \mathbf{H}_y^{n-1/2}) - \frac{\Delta t}{2\varepsilon_0}(\mathbf{J}_x^{n+1} + \mathbf{J}_x^n). \tag{4}$$

Eq. (3) can be written as

$$\frac{d\mathbf{J}_x}{dt} + \nu \mathbf{J}_x = \varepsilon_0 \omega_p^2 E_x, \tag{5}$$

if **E** is perpendicular to $\mathbf{B}_0$, the electron motion will be affected by $\mathbf{B}_0$. It turns out that waves with $\mathbf{E}\perp\mathbf{B}_0$ tend to be elliptically polarized instead of plane polarized. That is, as such a wave propagates into a plasma, it develops a component E_z along **k**, thus becoming partly longitudinal and partly transverse. To treat this mode properly, we must allow **E** to have both y and z components

$$\mathbf{E} = E_y \hat{y} + E_z \hat{z}. \tag{6}$$

The **E**-vector of an extraordinary wave is elliptically polarized. The components E_y and E_z oscillate 90° out of phase, so that the total electric field vector **E** has a tip that moves in an ellipse once in each wave period.

The dispersion relation for the extraordinary wave is

$$\frac{c^2 \mathbf{k}^2}{\omega^2} = 1 - \frac{\omega_p^2}{\omega^2} \frac{\omega^2 - \omega_p^2}{\omega^2 - \omega_h^2}, \tag{7}$$

where $\omega_h = \sqrt{\omega_p^2 + \omega_c^2}$ is the upper hybrid frequency.

As a wave propagates through a region in which ω_p and ω_c are changing, it may encounter cutoffs and resonances. The resonance of the extraordinary wave is found by setting **k** equal to infinity in Eq. (7). So that a resonance occurs at a point in the plasma where $\omega = \omega_h$. The cutoffs of the extraordinary wave are found by setting **k** equal to zero in Eq. (7), we obtain

$$\omega_\pm = \frac{1}{2}\left[(\omega_c^2 + 4\omega_p^2)^{1/2} \pm \omega_c\right], \tag{8}$$

where the subscripts + and − stand for right-hand and left-hand cutoffs. A wave is generally reflected at a cutoff and absorbed at a resonance. The reflection is large in the nonpropagation regions of $\omega < \omega$ and $\omega_h < \omega < \omega$; the transmission coefficient is large in the propagation regions of $\omega < \omega < \omega_h$ and $\omega > \omega_\pm$. The strong absorption occurs at the resonance point of the upper hybrid frequency, namely, $\omega = \omega_h$.

In this case, following equations can be derived from Eq. (1) using the Yee grid and leap-frog integration [1]

$$E_y^{n+1} = E_y^n + \frac{\Delta t}{\varepsilon_0 \Delta z}(\mathbf{H}_x^{n+1/2} - \mathbf{H}_x^{n-1/2}) - \frac{\Delta t}{2\varepsilon_0}(\mathbf{J}_y^{n+1} + \mathbf{J}_y^n), \tag{9}$$

$$E_z^{n+1} = E_z^n - \frac{\Delta t}{2\varepsilon_0}(\mathbf{J}_z^{n+1} + \mathbf{J}_z^n). \tag{10}$$

Eq. (3) can be written as

$$\frac{d\mathbf{J}_y}{dt} + \nu \mathbf{J}_y = \varepsilon_0 \omega_p^2 E_y - \boldsymbol{\omega}_b \mathbf{J}_z, \tag{11}$$

$$\frac{d\mathbf{J}_z}{dt} + \nu \mathbf{J}_z = \varepsilon_0 \omega_p^2 E_z + \boldsymbol{\omega}_b \mathbf{J}_y. \tag{12}$$

To derive the RKETD scheme [13], multiplying (5) through by the integrating factor $e^{\nu t}$, letting $t_{n+1} = t_n + \Delta t$, $t_{n+1} = t_n + \tau$. Then integrating the equation over a single time step from t_n to t_{n+1}, to give

$$\mathbf{J}_x^{n+1} = e^{-\nu\Delta t}\mathbf{J}_x^n + e^{-\nu\Delta t}\int_0^{\Delta t} e^{\nu\tau} F(t_n + \tau)\, d\tau, \tag{13}$$

where $F(t_n + \tau) = \varepsilon_0 \omega_p^2 E_x(t_n + \tau)$.

This formula is exact, and the essence of the RKETD methods is in deriving approximations to the integral in this expression. The first step is taken to give

$$K = e^{-\nu\Delta t}\mathbf{J}_x^n + \frac{F(t_n, \mathbf{J}_x)(1 - e^{-\nu\Delta t})}{\nu}. \tag{14}$$

Then the approximation

$$F(t_n + \tau) = F(t_n, \mathbf{J}_x) + \frac{\tau}{\Delta t}[F(t_n + \Delta t, K) - F(t_n, \mathbf{J}_x)] + o((\Delta t)^2). \tag{15}$$

By substituting Eq. (15) into Eq. (13), it yields

$$\mathbf{J}_x^{n+1} = e^{-\nu\Delta t}\mathbf{J}_x^n + F(t_n, \mathbf{J}_x) e^{-\nu\Delta t}\int_0^{\Delta t} e^{\nu\tau} d\tau + [F(t_n + \Delta t, K) - F(t_n, \mathbf{J}_x)]\frac{e^{-\nu\Delta t}}{\Delta t}\int_0^{\Delta t} \tau e^{\nu\tau}\, d\tau \tag{16}$$

After some integral manipulation the component of $\mathbf{J}_x$ at $n+1$ time step can be written as

$$\mathbf{J}_x^{n+1} = e^{-\nu\Delta t}\mathbf{J}_x^n + \frac{(1 - e^{-\nu\Delta t})}{\nu}\varepsilon_0 \omega_p^2 E_x^n + \frac{(e^{-\nu\Delta t} - 1 + \nu\Delta t)}{\nu^2 \Delta t}[\varepsilon_0 \omega_p^2 E_x^{n+1} - \varepsilon_0 \omega_p^2 E_x^n]. \tag{17}$$