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Birefringent network forming a rotator

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ABSTRACT

An optical system consisting of a large number N of identical retardation plates, where each of the plates is rotated by an angle θ with respect to the one preceding it, has been examined by Reusch and Sohncke. The product of N and θ must be equal to $m\pi$, where m is an integer. This system behaves equivalent to a simple optical retarder under certain conditions. The system, which is quite difficult to examine by ordinary methods, has been treated using Jones calculus by Jones himself. But the treatment seems to be still quite complicated. We have treated the same system using Jones calculus, Pauli's spin matrices and Chebyshev polynomial which gives the treatment quite elegant formalism.

1. Introduction

The optical properties of a system composed of N identical retardation plates, whose axes are distributed uniformly about a whole number of semicircles, had been examined first by Reusch [1]. If the number of semicircles are denoted by m, then we may say that the axis of each plate is rotated through an angle $\theta = m\pi/N$ with respect to the corresponding axis of the plate immediately preceding it. Reusch experimentally found that if the phase retardation value of each of the plates is small, the complete optical system behaves approximately equivalent to a simple rotator. After Reusch published his results, Sohncke [2] provided a rigorous mathematical treatment of the system composed of three retardation plates with their axes distributed through a semicircle, which means the relative angle was $\pi/3$. Pockels [3] also derived an approximate expression for the angle of the rotation produced by a system of N retardation plates distributed through a semicircle. Then Jones [4] presented a rigorous treatment of the same optical system of identical N retardation plates. He used the transformation properties of matrices to find an exact expression for the matrix M^N of the entire optical system.

A rigorous treatment of the same optical system using Chebyshev polynomial for getting an exact expression for the matrix M^N of the entire system has been presented here. The current treatment provides much simpler technique compared to that of Jones' to find the equivalent matrix of the system. The optical system under consideration contains only retardation plates. Hence the system may be replaced by a system composed only of one rotator and one retardation plate [5]. The exact and reasonably simple expressions for the angle ϕ of the equivalent rotator and the phase retardation δ of the equivalent retardation plate have also been investigated. The condition which is to be satisfied in order that the optical system behaves approximately as a rotator has also been derived. The results are exactly the same as that of Jones' result and confirm Pockels' approximate result.

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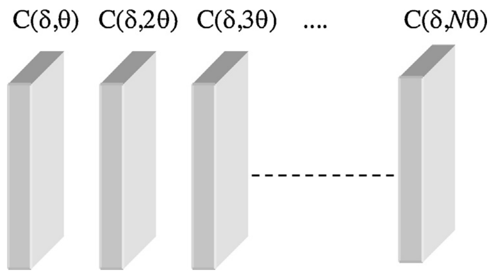

Fig. 1. Schematic diagram of the optical system.

2. Derivation of the overall matrix

The optical system under consideration is illustrated in Fig. 1.

We consider the phase retardation of the individual plates to be δ. Hence the matrix of the complete optical system is

$$\begin{aligned} M^N &= \mathrm{R}\left(N\theta\right)\mathrm{C}\left(\delta\right)\mathrm{R}\left(-N\theta\right)\mathrm{R}\left(\overline{N-1}\theta\right)\cdots\mathrm{R}\left(2\theta\right)\mathrm{C}\left(\delta\right)\mathrm{R}\left(-2\theta\right)\mathrm{R}\left(\theta\right)\mathrm{C}\left(\delta\right)\mathrm{R}\left(-\theta\right) \\ &= \mathrm{R}(m\pi)\left[C\left(\delta\right)R\left(-\theta\right)\right]^N \\ &= (-1)^m\left[C\left(\delta\right)R\left(-\theta\right)\right]^N \end{aligned} \tag{1}$$

where N is the number of retarders having retardation δ, $C(\delta)$ and $R(-\theta)$ are the relevant Jones matrices of the elements of the system given by

$$C\left(\delta\right) = \begin{vmatrix} \exp\left(i\delta/2\right) & 0 \\ 0 & \exp\left(-i\delta/2\right) \end{vmatrix} \tag{2}$$

and

$$R\left(-\theta\right) = \begin{vmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{vmatrix} \tag{3}$$

Let us assume

$$C\left(\delta\right)R\left(-\theta\right) = T\left(\delta,\theta\right) \tag{4}$$

Substituting $T(\delta,\theta)$ for the product $C(\delta)R(-\theta)$ in the above Eq. (1), the Jones vector of the complete system may be written as

$$M^N = (-1)^m\left[T\left(\delta,\theta\right)\right]^N \tag{5}$$

Carrying out the matrix multiplications, $T(\delta,\theta)$ is found to be a unimodular matrix. Now, we raise the matrix $T(\delta,\theta)$ to the N^{th} power, using a well known property of a unimodular matrix. Thus we get,

$$\left[T\left(\delta,\theta\right)\right]^N = \begin{vmatrix} m_{11}P_{N_1-1}(x_1) - P_{N_1-2}(x_1) & m_{12}P_{N_1-1}(x_1) \\ m_{21}P_{N_1-1}(x_1) & m_{22}P_{N_1-1}(x_1) - P_{N_1-2}(x_1) \end{vmatrix} \tag{6}$$

where m_{ij} is the element in the ith row and jth column of the matrix $M(\delta,\theta)$, and say

$$x = (m_{11}+m_{22})/2 = \cos\theta\cos\delta/2 = \cos\chi(\text{say}) \tag{7}$$

$$\therefore \chi = \cos^{-1}x \text{and} \left(1-x^2\right)^{1/2} = \sin\chi \tag{8}$$

here, P_Ns are the Chebyshev polynomials of the second kind, given by

$$P_N(x) = \sin(N+1)\cos^{-1}x/\left(1-x^2\right)^{1/2} \tag{9}$$

Now carrying out the operations on the right hand side of Eq. (6), we obtain

$$\left[T\left(\delta,\theta\right)\right]^N = \begin{vmatrix} \cos N\chi + i\sin N\chi\dfrac{\cos\theta\sin\dfrac{\delta}{2}}{\sin\chi} & \dfrac{\sin N\chi}{\sin\chi}\sin\theta\exp\left(i\delta/2\right) \\ -\dfrac{\sin N\chi}{\sin\chi}\sin\theta\exp\left(-i\delta/2\right) & \cos N\chi - i\sin N\chi\dfrac{\cos\theta\sin\dfrac{\delta}{2}}{\sin\chi} \end{vmatrix} \tag{10}$$