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Hybrid synchronization and parameter identification of uncertain interacted networks

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ABSTRACT

In this paper, we study hybrid synchronization and parameter identification of uncertain interacted networks and investigate inner synchronization and outer synchronization between these two networks. Based on the linear matrix inequality (LMI), we obtain the synchronous conditions for the inner synchronization in the form of LMIs. When the node dynamical equation has an unknown parameter vector, we design the appropriate controllers to realize the outer synchronization and simultaneously identify the unknown parameter. Finally, we provide two numerical examples to show the efficiency of the obtained theoretical results.

1. Introduction

When the small-world [1] and scale-free [2] networks were proposed, the studies on complex networks have been a hot topic in the research community, such as physics, mathematics, computer science and engineering disciplines [3–5]. Diverse models on complex networks have been put forward, including the traffic networks, the science citation network, metabolic networks, and fractal networks [6], etc. Generally average path length, clustering coefficient, degree distribution and betweenness are most used for measuring the complex networks. Among of the dynamics on the complex networks, synchronization is an interesting issue with an aim of considering the effect of node dynamics and topological structures on the synchronization [7,8], on the other hand, when the synchronization is not achieved by the appropriate node dynamics and topology, we often apply the controlling methods (e.g., the adaptive, pinning, and impulsive) to realize the synchronization [9–15] and many references cited therein.

Apart from the inner synchronization of complex networks, there exist other types of synchronization, such as outer synchronization [16,17], lag synchronization [18], and anti-synchronization [19]. For the outer synchronization, the research goal is to study the collective dynamics between two coupled networks as a special example of multi-layer network [20]. The interactions between two networks are colorful, e.g., unidirectional, bidirectional, and mutual couplings. When the outer synchronization does not happen under the corresponding interactions, the controllers are often used. Based on the original work [16], Tang et al. [21] first designed the adaptive controllers to realize the outer synchronization between two nonidentical networks. Afterwards, Sun et al. studied mixed synchronization between two coupled networks in [22], including the inner synchronization and outer synchronization, however it does not involve with the parameter identification. In addition, outer synchronization between two networks has been investigated in [23] with a feature that the node dynamical equation is described by the fractional order function.

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In the above-mentioned works on the outer synchronization [16,21,22], the node dynamical equations are all given in advance. Recently synchronization of uncertain coupled networks has gained considerable attention. Uncertain networks refer that there exist some unknown information in the networks, e.g., the unknown parameter vector and the coupling structures. An interesting issue is to identify the unknown quantities based on the synchronization. In [24], Sun and Li used the nonlinear controllers to achieve the generalized outer synchronization between two networks where the coupling structures are described by the nonlinear functions. Wu and Lu [25] studied the outer synchronization and parameter identification between two networks with time-varying connections by designing the adaptive controllers. The adaptive projective lag synchronization of uncertain complex dynamical networks with delayed coupling was investigated in [26]. Wu and Liu [27] studied the outer synchronization and parameter identification between two coupled networks where the time delay exists in the node dynamics.

Inspired by the above discussions, we investigate the hybrid synchronization of two interacted networks, including the inner synchronization inside each network and parameter identification based on outer synchronization by the designed controllers. Through the tool of linear matrix inequality, we obtain a theorem on the inner synchronization inside each network and design the appropriate controllers to realize the outer synchronization. When the outer synchronization is achieved, the unknown parameter is exactly identified. Finally we give numerical examples to show the effectiveness of our obtained results.

The remainder of this paper is organized as follows. The network models and mathematical preliminaries are given in Section 2. Section 3 gives the analysis on the inner and outer synchronization. Numerical examples are provided in Section 4. Conclusions and remarks are included in Section 5.

Notations: Throughout this paper, for a symmetric matrix A, the notation $A>0$ ($A<0$) means that the matrix A is positive definite (negative definite). The norm of a vector x is $\|x\| = \sqrt{x^T x}$. $\otimes$ is the kronecker product. $\lambda_{\max}(A)$ is the maximal eigenvalue of matrix A. I_n is an identity matrix of size n.

2. Preliminaries and model presentation

The dynamical model on the uncertain interacted networks is given as follows:

$$\dot{x}_i(t) = f(x_i(t)) + \sum_{j=1}^{N} a_{ij}\Gamma(y_j(t) - x_j(t)), \quad i = 1, 2, \ldots, N, \tag{1}$$

$$\dot{y}_i(t) = f(y_i(t)) + \sum_{j=1}^{N} b_{ij}\Gamma(x_j(t) - y_j(t)), \quad i = 1, 2, \ldots, N, \tag{2}$$

where $x_i, y_i \in R^n$ are the state vectors. The node dynamical functions: $f(\cdot)$: $R^n \to R^n$ and $g(\cdot)$: $R^n \to R^n$ are continuously differential. The matrix $\Gamma \in \Re^{n\times n}$ is a constant 0-1 matrix linking coupled variables. For simplicity, one often supposes that $\Gamma = \mathrm{diag}(r_1, r_2, \ldots, r_n)$ is a positive definite matrix. $A = (a_{ij})_{N\times N}$ and $B = (b_{ij})_{N\times N}$ denote the interacted coupling matrices between these two networks, whose elements a_{ij} are the intensity from i in network X to j in network Y, similarly the entries of B are same defined as A.

Assumption 1. There exists a positive constant L satisfying

$$(y-x)^T(f(y,t) - f(x,t)) \le L(y-x)^T(y-x),$$

where $x, y \in R^n$ are time-varying vectors.

Lemma 1. *((LaSalle invariance principle) [28]). Let $\Omega \subset D$ be a compact set that is positively invariant with respect to $\dot{x} = f(x)$. Let V: $D \to R$ be a continuously differentiable function such that $\dot{V}(x) \le 0$ in Ω. Let E be the set of all points in Ω where $\dot{V}(x) = 0$. Let M be the largest invariant set in E. Then every solution starting in Ω approaches M as $t \to \infty$.*

Definition 1. Inner and outer synchronization between networks (1) and (2) are achieved if

$$\lim_{t\to\infty}\|x_i(t) - x_s(t)\| = 0; \qquad \lim_{t\to\infty}\|y_i(t) - y_s(t)\| = 0; \qquad \lim_{t\to\infty}\|y_i(t) - x_i(t)\| = 0, \quad i = 1, \ldots, N,$$

where $x_s(t)$, $y_s(t)$ are the synchronized states of networks X and Y.