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# A new fractional Biswas–Milovic model with its periodic soliton solutions

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ABSTRACT

The present study introduces a general form of fractional Biswas–Milovic (FBM) equation. After this, some exact solutions for FBM equation are obtained by Exp-function and sine–cosine methods. This work illustrates the validity and great potential of the Exp-function and sine–cosine approaches for the nonlinear space–time fractional differential equations with complex valued solutions. One can see that the methods are relatively easy and efficient to use.



## 1. Introduction

Many important phenomena in science and engineering can be modeled by nonlinear partial differential equations (NLPDEs). NLPDEs are widely used to describe complex phenomena in various sciences, especially in physics and applied mathematics. Fractional differential operators are as old as the differential calculus. The idea of fractional differential goes back when L'Hopital tries to answer this equation: "What is the meaning of $d^n y/dx^n$ if $n=1/2$?". The theory of derivatives of fractional order have appeared in many fields of science and engineering and also become an important tool in mathematical modelling [1–5]. Fractional differential equations (FDEs) have been found to be effective to describe some physical phenomena such as damping law, rheology, diffusion processes, quantum, mechanics, fluid flow, finance, viscoelasticity, quantum, chemistry and so on. One of the best advantages of use of FDEs is modelling and control of many dynamic systems. In fact FDEs are powerful instruments to describe real world problems more accurately than the classical integer-order ones. Although fractional derivatives have a long mathematical history, for many years they were not used in physics. One reason for this unpopularity could be that there are multiple nonequivalent definitions of fractional derivatives. Also, the nonlocal property of fractional derivatives makes another difficulty of using this kind of differentiation. However, because of its applications, during the last decades fractional calculus starts to attract much more attention of physicists and mathematicians. Some FDEs are direct extensions of integer-order differential equations. Fractional KPP equation, fractional STO equation, fractional PKP equation, fractional SRLW equation, fractional Riccati equation, show some of these extensions. Although there are many analytical and numerical methods for solving classical integer-order differential equations, but usually we cannot use these methods to solve the relevant fractional order equations. Therefore, we have to present new schemes to solve FDEs. Some numerical methods which solve FDEs are: the differential transform method [6], enhanced

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Homotopy perturbation method [7], the new Homotopy perturbation method [8], the modified Homotopy perturbation method [9], the variational iteration method [10], the Adomian decomposition method [11], the Homotopy analysis method [12], Adams–Baskforth–Moultun method [13], and Haar wavelet operational matrix method [14]. However applications of some of these methods have limitations and disadvantages. For example, Adomian decomposition method has a complicated algorithm in computation of Adomian polynomials for nonlinear equations. In each iteration of Homotopy perturbation method, one must solve a linear functional equation which sometimes is not easy. The accuracy of Homotopy perturbation method is dependent on selection of the auxiliary parameter $\hbar$ of the zero-order deformation equation. Further, there is an inherent inaccuracy in variational iteration method due to identifying the Lagrange multiplier in the scheme. Besides, one cannot expect the solution of an FDE to be smooth and this poses a challenge to the convergence analysis of numerical schemes. Hence, searching for new methods which could obtain solutions for FDEs is an attractive research area. The importance of obtaining exact or approximate solutions of fractional differential equations in physics and mathematics, is still a hot spot to seek new methods to present exact or approximate solutions. Exact solutions to nonlinear equations help to understand the physical phenomena they describe in nature. Exact solutions for fractional differential equations play an important role in many phenomena in physics such as fluid mechanics, hydrodynamics, optics, plasma physics and so on. Recently new approaches to find analytical solutions of FDEs have been presented, such as extended tanh-method [15], modified Kudryashov method [16], the Exp-function method [17], sub-equation method [18], the $\exp(-\varphi(\xi))$-expansion method [19], the $(G'/G)$-expansion method [20], etc.

In this paper we have two goals. First, we introduce a general form of fractional Biswas–Milovic equation, which is a new equation. In fact, the fractional form of Biswas–Milovic equation is a new fractional form. Next, we obtain the exact solutions of FBM equation by Exp-function and sine–cosine methods. The Biswas–Milovic equation that is going to be studied in this paper is given by [21]

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = 0, \tag{1}$$

where $i = \sqrt{-1}$ and the dependent variable $q$ is a complex valued function, while $x$ and $t$ are two independent variables. The coefficients $a$ and $b$ are constants where $ab > 0$, also $m \geq 1$ is a parameter.

In fact, Eq. (1) is the dimensionless form of nonlinear Schrödinger's equation arises in the study of long-distance optical communications and all-optical oltra fast switching devices. This equation has been indicated to manage the evolution of a wave packet in a weakly nonlinear and dispersive medium and has eventuated diverse fields such as nonlinear optics, water waves and plasma. Eq. (1) is a nonlinear partial differential equation that is not integrable, in general. The non-integrability is not necessarily related to the nonlinear term in it. Also, in (1), F is a real-valued algebraic function and it is necessary to have smoothness of the complex function $\mathrm{F}(|q|^2) : \mathbb{C} \mapsto \mathbb{C}$. Considering the complex plane $\mathbb{C}$ as a two-dimensional linear space $\mathbb{R}^2$, the function $\mathrm{F}(|q|^2)$ is $k$ times continuously differentiable, so that [22]

$$\mathrm{F}(|q|^2) \in \bigcup_{l,n=1}^{\infty} \mathbb{C}^k((-n, n) \times (-l, l); \quad \mathbb{R}^2). \tag{2}$$

In order to seek exact solutions of Eq. (1), one assumes that $q(x, 0) = \mathrm{e}^{ix}$ and in this case the Kerr law of nonlinearity which appears in nonlinear optics [23] is

$$F(s) = s, \tag{3}$$

so Eq. (1) becomes

$$i(q^m)_t + a(q^m)_{xx} + b(|q|^2)q^m = 0. \tag{4}$$

The outline of this paper is organized as follows. In the following section we present a review on the basic definitions about fractional derivatives. The Exp-function and sine–cosine methods are briefly reviewed in Section 3. In Section 4 after introducing of fractional form of BM equation, some exact solutions of fractional BM equation are presented. In Section 5, we conclude our results, give some comments and discussions.

## 2. Preliminaries

There are several different definitions of the concept of a fractional derivative [5]. Some of these are Riemann–Liouville, Grunwald–Letnikow, Caputo, and modified Riemann–Liouville derivatives. The most commonly used definitions are the Riemann–Liouville and Caputo derivatives. The Jumarie's modified Riemann–Liouville derivative [24] of order $\alpha$ is given by the following expression:

$$D_t^{\alpha} f(t) = \begin{cases} \dfrac{1}{\Gamma(1-\alpha)} \dfrac{d}{dt} \displaystyle\int_0^t (t-\xi)^{-\alpha} (f(\xi) - f(0)) d\xi, & 0 < \alpha < 1, \\ (f^{(n)}(t))^{(\alpha-n)}, & n \leq \alpha < n, n \geq 1. \end{cases}$$