Optik 127 (2016) 7842–7851

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Propagation properties of flat-topped vortex hollow beam in uniaxial crystals orthogonal to the optical axis

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ARTICLE INFO

Article history:
Received 6 April 2016
Accepted 31 May 2016

Keywords:
Flat-topped vortex hollow beam
Uniaxial crystals
Laser propagation
Intensity properties

ABSTRACT

The analytical expressions of a flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis are obtained. The evolution properties of intensity and phase for flat-topped vortex hollow beam are discussed by using the numerical examples in detail. It is found that the beam order N, topological charge M and the ratio of refractive index n_e/n_o of uniaxial crystals will influent the beam intensity and phase distributions.

1. Introduction

In the past years, the uniaxial crystal have been widely studied in the application of wave plates, polarizer, compensator, and optical modulation devices [1]. Thus the propagation properties of optical beams in uniaxial crystals have been widely studied. Based on the vectorial theory of optical beam propagating in uniaxial crystals orthogonal to the optical axis [2], the evolution properties of various optical beams in uniaxial crystal have been investigated, such as Laguerre-Gauss and Bessel-Gauss beams [3], dark hollow beams [4], flat-topped beams [5–7], beams generated by Gaussian mirror resonator in uniaxial crystals [8], partially polarized and partially coherent beam [9], higher-order cosh-Gaussian beams [10], Laguerre-Gaussian correlated Schell model beam [11], elliptical Gaussian vortex beam [12], four-petal Gaussian vortex beam [13,14], et al.

With the development of laser optics, more and more optical vortex beams have been introduced and investigated in detail. Among the optical vortex beams introduced by the researchers, the evolution properties of optical vortex beams, such as elliptical Gaussian vortex beam [12], partially coherent four-petal Gaussian vortex beam [13], four-petal Gaussian vortex beam [14,15], vortex beam carried by an Airy beam [16] et al., have been studied. However, to our knowledge, there have no reports about the flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis. In this work, we mainly investigate the paraxial propagation properties of flat-topped vortex hollow beams in uniaxial crystals orthogonal to the optical axis, and study the influences of extraordinary to ordinary refractive indices, the beam parameters M and N on the intensity and phase distributions of flat-topped vortex hollow beams.

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http://dx.doi.org/10.1016/j.ijleo.2016.05.155
0030-4026/

2. Theoretical analysis

When the optical beam propagating in uniaxial crystals orthogonal to the optical axis, assume that optical axis of uniaxial crystals coincides with the x-axis, and the optical beam propagates along the z-axis, by using the paraxial approximation, the components of the optical beams propagating in uniaxial crystals orthogonal to the optical axis can be written as [2]:

$$E_x(\mathbf{r},z)=\exp(ikn_ez)\frac{kn_o}{2\pi iz}\times\int dx_0dy_0\exp\left\{-\frac{k}{2izn_e}\left[n_o^2(x-x_0)^2+n_e^2(y-y_0)^2\right]\right\}E_x(\mathbf{r}_0,0) \tag{1a}$$

$$E_y(\mathbf{r},z)=\exp(ikn_oz)\frac{kn_o}{2\pi iz}\times\int dx_0dy_0\exp\left\{-\frac{kn_o}{2iz}\left[(x-x_0)^2+(y-y_0)^2\right]\right\}E_y(\mathbf{r}_0,0) \tag{1b}$$

where $k=2\pi/\lambda$ is the wave number; $\mathbf{r}=(x,y)$ and $\mathbf{r}_0=(x_0,y_0)$ are the position vectors at the output plane and input plane, respectively; $E_\alpha(\mathbf{r}_0,0)$ and $E_\alpha(\mathbf{r},z)$ $\alpha=(x,y)$ are the electric fields at the input plane and output plane. It can be found that the x component of the optical beams propagating in uniaxial crystals undergoes diffraction spreading asymmetry in x-y plane, and the y component is the same as which propagating in isotropic media. In this work, the x-polarized flat-topped vortex hollow beam propagating in uniaxial crystals orthogonal to the optical axis is analyzed. The x polarized flat-topped vortex hollow beam propagating in uniaxial crystals can be expressed as [17]:

$$E(\mathbf{r}_0,0)=\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n}\exp\left[-n\left(\frac{x_0^2}{w_x^2}+\frac{y_0^2}{w_y^2}\right)\right]\left(\frac{x_0}{w_x}+i\frac{y_0}{w_y}\right)^M \tag{2}$$

where N is the order of the flat-topped vortex hollow beam, M is the topological charge, w_x and w_y are the beam waist width at x and y direction, $\binom{N}{n}$ denotes the binomial coefficient.

By recalling the following integral equations [18]:

$$(x+iy)^M=\sum_{s=0}^{M}\frac{M!i^s}{s!(M-s)!}x^{M-s}y^s \tag{3}$$

$$\int_{-\infty}^{+\infty}x^n\exp\left(-px^2+2qx\right)dx=n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[\frac{n}{2}]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \tag{4}$$

$$H_n(l)=\sum_{k=0}^{[\frac{n}{2}]}\frac{(-1)^kn!}{k!(n-2k)!}(2l)^{n-2k} \tag{5}$$

Substituting the Eq. (2) of flat-topped vortex hollow beam into Eq. (1a), we obtain

$$\begin{aligned}E_x(\mathbf{r},z)=&\exp(ikn_ez)\frac{\pi kn_o}{2\pi iz}\exp\left(-\frac{kn_o^2}{2izn_e}x^2-\frac{kn_e}{2iz}y^2\right)\\&\times\sum_{n=1}^{N}\frac{(-1)^{n-1}}{N}\binom{N}{n}\sum_{l=0}^{M}\frac{M!i^l}{s!(M-s)!}\left(\frac{1}{w_x}\right)^{M-l}\left(\frac{1}{w_y}\right)^l\\&\times\sqrt{\frac{\pi}{a_x}}2^{-M+l}i^{M-l}\left(\frac{1}{\sqrt{a_x}}\right)^{M-l}\exp\left(\frac{c_x^2}{a_x}x^2\right)H_{M+l}\left(-\frac{ic_x}{\sqrt{a_x}}x\right)\\&\times\sqrt{\frac{\pi}{a_y}}2^{-M+l}i^{M-l}\left(\frac{1}{\sqrt{a_y}}\right)^{M-l}\exp\left(\frac{c_y^2}{a_y}y^2\right)H_{M+l}\left(-\frac{ic_y}{\sqrt{a_x}}y\right)\end{aligned} \tag{6}$$

with

$$a_x=\frac{n}{w_x^2}+\frac{kn_o^2}{2izn_e} \tag{7a}$$

$$c_x=\frac{kn_o^2}{2izn_e} \tag{7b}$$