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Multiple attractors and dynamic analysis of a no-equilibrium chaotic system

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ABSTRACT

The hidden attractor holds attracting basin that does not intersect with any small neighborhoods of equilibrium. The attractors of chaotic systems with no-equilibrium, only stable equilibrium or a line of equilibrium belong to hidden attractors, and in which the Shilnikov method is not suitable to explain the chaos. This paper introduces a three-dimensional chaotic system with no equilibrium point. The fundamental dynamical properties as phase portrait, power spectral density, state trajectory, Lyapunov exponent, Kaplan-Yorke dimension and bifurcation are displayed and discussed, which show that the new system exhibits rich dynamics and can generate multiple attractors with different system parameters and initial values.

1. Introduction

Chaos of nonlinear dynamical system has some special properties as boundedness, sensitive to initial condition, dense periodic orbit and infinite recurrence. Chaotic behave exists widely in different scientific disciplines such as biology, chemical reaction, economy, electrical circuit, neural network, etc [1–5]. Since the significant chaotic system of a weather model is found by Lorenz [6], many new chaotic systems have been reported for the potential technological applications [7–13].

The equilibrium point of a chaotic system plays an important role on investigating its dynamical behavior. Most introduced chaotic systems, such as Lorenz system [6], Chen system [7], Deng system [9], Li-Wu system [10], Rossler system [14], etc., have several equilibrium points. These systems hold at least one unstable equilibrium point, and the existence of chaos can be verified based on Shilnikov criteria [15]. Thus it brings on four classes of chaos: heteroclinic orbit chaos, homoclinic orbit chaos, hybrid type with both heteroclinic and homoclinic orbits, and chaos without heteroclinic orbit and homoclinic orbit. It deserved to recall that a heteroclinic orbit is a trajectory that connects two equilibrium points, or connects a closed orbit to another one, and a homoclinic orbit is a trajectory that is doubly asymptotic to an equilibrium point, or closed orbit asymptotic to itself.

Recently, a new classification of chaotic attractors is introduced as hidden attractor and self-excited attractor based on the simplicity of finding basin of attraction in the phase space [16,17]. For hidden attractor, the attracting basin does not intersect with any small neighborhoods of its equilibrium point, whereas the self-excited attractor holds a basin of attraction associated with an unstable equilibrium. Therefore, the simplest examples of hidden attractors are such systems with no-equilibrium, only stable equilibrium or a line of equilibrium points [18–23]. Up to now, there is little knowledge

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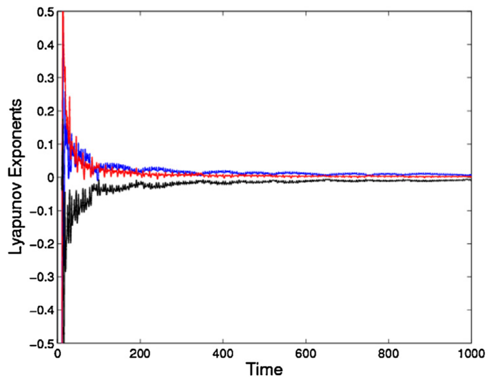

Fig. 1. Lyapunov exponents of system (1).

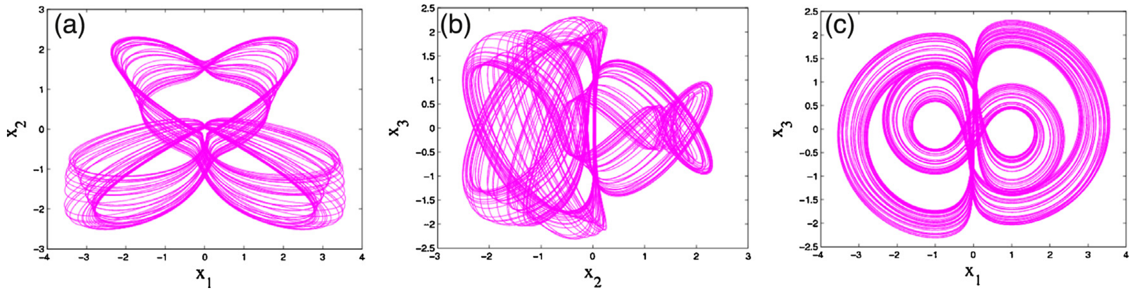

Fig. 2. Chaotic phase diagrams of system (1). (a) x_1– x_2 plane; (b) x_2– x_3 plane; (c) x_1–x_3 plane.

about the dynamical behavior of these systems, and the Shilnikov criteria cannot be adopted to prove the chaos for the lack of heteroclinic or homoclinic orbit [22,23]. Thereby, it further reveals that it is certainly not necessary to hold at least one unstable equilibrium point for generating chaos.

In this paper, we introduce a three-dimensional chaotic system with no equilibrium point. The Shilnikov method is not applicable to verify the chaos of this system because there is no equilibrium point and thus there is no heteroclinic or homoclinic orbit. The fundamental dynamical properties as phase portrait, power spectral density, state trajectory, Lyapunov exponent, Kaplan-Yorke dimension and bifurcation are displayed and discussed, which show that the new system exhibits rich dynamics and can generate multiple attractors with different system parameters and initial values.

2. The introduced chaotic system

The reported 3D chaotic system consists of two quadratic cross-product terms and a real constant term, depicted as

$$\begin{cases} \dot{x}_1 = ax_2 + bx_1x_3 \\ \dot{x}_2 = -cx_1 + dx_2x_3 \\ \dot{x}_3 = 1 - ex_1{}^2 \end{cases} \tag{1}$$

The parameters a, b, c, d, e of system (1) are positive. When we set $a = 0.5$, $b = 1.9$, $c = 1$, $d = 1$, $e = 1$ and initial condition (0.01, 0.05, −0.03), the Lyapunov exponents are calculated as 0.069825, 0.00072635, −0.09551, described in Fig. 1. The Kaplan-Yorke dimension is calculated as $D_{KY} = 2 + (0.069825 + 0.00072635)/0.09551 = 2.7387$. Therefore, system (1) is really chaotic with fractional dimension. The corresponding chaotic phase diagrams are shown in Fig. 2. It's known from Fig. 2 that the introduced system displays complicated dynamical behaviors.

The corresponding random time trajectories and continuous broadband frequency spectrum further reveal the complicated dynamical behavior of system (1), displayed in Fig. 3.

3. Analysis of equilibrium point

The equilibrium point of system (1) is gotten by the condition $\dot{x}_1 = 0, \dot{x}_2 = 0, \dot{x}_3 = 0$. From $\dot{x}_1 = 0, \dot{x}_2 = 0$, we obtain

$$\frac{c}{d}x_1{}^2 + \frac{a}{b}x_2{}^2 = 0 \tag{2}$$