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Temporal power spectra of plane wave considering finite turbulence inner and outer scales in anisotropic turbulence

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ABSTRACT

Theoretical investigations and experiments have shown that the atmosphere turbulence exhibits both anisotropic and non-Kolmogorov properties compared with traditional atmospheric turbulence which takes the hypotheses of isotropy and homogeneity. There is an increasing interest for modeling atmosphere turbulence with a power spectrum that incorporates anisotropy and general spectral power law deviated from the standard value of 11/3 for the Kolmogorov turbulence. In this study, new analytic expressions for the temporal power spectra of irradiance fluctuations and angle of arrival (AOA) fluctuations have been derived based on the Rytov approximation theory for optical plane wave propagating through weak anisotropic non-Kolmogorov atmosphere turbulence. Compared with the previously published results, the effective anisotropic factor is introduced to the final results to include the anisotropy for different turbulence cells or eddies. The finite turbulence inner and outer scales and the general spectral power law in the range of 3–4 are also considered in the final derived models. The results in this work will help better understand the optical waves' propagation through anisotropic non-Kolmogorov turbulence.

1. Introduction

The turbulence effects of optical waves under atmospheric turbulence are produced by the random fluctuations of atmosphere refractive-index, and they are described by both spatial and temporal statistical models. With Taylor's frozen hypothesis theory, these two kinds of statistical models are interconvertible. If the average wind speed that is transverse to the propagation path is known, the spatial statistical models of turbulence effects at different spatial positions can be obtained indirectly from the temporal statistical ones which are convenient to be detected by experiments. Temporal power spectrum of an optical wave is an important model to describe the temporal statistical property of turbulence effects, and its theoretical investigation attracts more and more attentions [1–3]. They mainly focused on the isotropic non-Kolmogorov turbulence which takes a general spectral power law value in the range 3–4 instead of the standard value of 11/3 for the Kolmogorov turbulence and assumes the turbulence is isotropic (turbulence cell or eddy scale is the same in both horizontal and vertical directions). However, experiments and theoretical results have shown that the atmospheric turbulence also exhibits anisotropic and non-Kolmogorov properties [4–17]. They observed that the anisotropy is usually present at high altitude, above the atmospheric boundary layer, which extends to about 2 km in altitude and it is more evident for large turbulence cells or eddies [16]. Anisotropy can be present also at few meters above the ground [4]. Compared with the isotropic turbulence, the anisotropic turbulence cells or eddies do not take the same sizes in both horizontal and vertical

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propagation paths. Commonly, a horizontal turbulence cell or eddy scale is bigger than the vertical one, and it may lead to different statistical properties for optical waves propagating in different directions.

To investigate theoretically the optical waves' propagation through anisotropic non-Kolmogorov atmospheric turbulence media, the anisotropic non-Kolmogorov turbulence refractive-index fluctuations spectrum which adopted the circular symmetric assumption of turbulence cells or eddies in the orthogonal xy-plane throughout the path [14,16] was first derived and applied in the modeling of turbulence effects of optical waves [16–18]. The circular symmetric assumption is a special case of the distribution of turbulence cells or eddies in the orthogonal xy-plane. Then, to consider the asymmetry property of turbulence cells or eddies in the orthogonal xy-plane throughout the path, the more general anisotropic non-Kolmogorov turbulence refractive-index fluctuations spectrum was proposed [19,20]. This more general spectrum may lead to different statistical values in the horizontal and vertical transverse directions. But, these two spectral models are only valid in inertial subrange and the finite turbulence inner and outer scales are not considered. In the theoretical investigations, they are extended to the whole range by assuming zero turbulence inner scale and infinite turbulence outer scale. To address this problem, Toselli [21] introduced the finite turbulence inner and outer scales into the anisotropic turbulence refractive-index fluctuations spectrum and the concept of anisotropy at different turbulence cell or eddy scales has also been introduced by defining an effective anisotropic parameter with two specific cases of linear and parabolic anisotropic laws. Based on this spectral model, new analytic expressions for the temporal power spectra of irradiance fluctuations and AOA fluctuations will be derived in this work for optical plane wave propagating through weak anisotropic non-Kolmogorov turbulence. The effective anisotropic factor which describes the effect of anisotropy at different turbulence cell or eddy scales, the general spectral power law value and the finite turbulence inner and outer scales will be considered simultaneously in the derived models. Calculations will be performed to analyze the derived temporal power spectra.

2. Anisotropic turbulence refractive-index fluctuations spectrum with finite turbulence inner and outer scales

In this work, the anisotropic turbulence refractive-index fluctuations spectrum reported in [21] is introduced to investigate theoretically the temporal power spectra of plane wave propagating through anisotropic non-Kolmogorov turbulence. This refractive-index fluctuations spectrum considers simultaneously the effective anisotropy factor ς_{eff}, the finite turbulence inner and outer scales (l_0 is the turbulence inner scale and L_0 is the turbulence outer scale), and the general spectral power law α. It takes the form as [21]:

$$\Phi_n\left(\kappa,\alpha,\varsigma_{eff}\right)=A(\alpha)\times\hat{C}_n^2\times\ \varsigma_{eff}^2\times\left(\varsigma_{eff}^2\kappa_{xy}^2+\kappa_z^2+\kappa_0^2\right)^{-\frac{\alpha}{2}}\exp\left(-\frac{\varsigma_{eff}^2\kappa_{xy}^2+\kappa_z^2}{\kappa_m^2}\right),\quad\left(\kappa>0,\quad 3<\alpha<4\right). \tag{1}$$

$$A(\alpha)=\frac{1}{4\pi^2}\Gamma(\alpha-1)\cos\left[\frac{\alpha\pi}{2}\right]. \tag{2}$$

in which, κ is the wavenumber related to the turbulence cell or eddy size, $\kappa=\sqrt{\varsigma^2\left(\kappa_x^2+\kappa_y^2\right)+\kappa_z^2}=\sqrt{\varsigma^2\kappa_{xy}^2+\kappa_z^2}$, κ_x, κ_y, and κ_z are the components of κ in x, y, and z directions. $\kappa_0=2\pi/L_0$ and $\kappa_m=c(\alpha)/l_0$.

$$c(\alpha)=\left\{\pi A(\alpha)\Gamma\left(\frac{3}{2}-\frac{\alpha}{2}\right)\left(\frac{3-\alpha}{3}\right)\right\}^{\frac{1}{\alpha-5}}. \tag{3}$$

Where, $\hat{C}_n^2=\beta C_n^2$ is the generalized structure parameter with unit $[m^{3-\alpha}]$ and β is a dimensional constant with unit $[m^{11/3-\alpha}]$. For Kolmogorov turbulence ($\alpha=11/3$), the generalized structure parameter reduces to the structure parameter C_n^2 with unit $[m^{-2/3}]$. $\Gamma(\cdot)$ is the gamma function.

This anisotropic turbulence refractive-index fluctuations spectral model is basically a generalized von Karman power spectrum with an anisotropic factor ς_{eff} which introduces the turbulence rescaling due to anisotropy along the z direction. In particular, when the turbulence inner and outer scales are set separately to zero and infinity, the spectrum shown in Eq. (1) reduces correctly to that in [14].

3. Temporal power spectrum of irradiance fluctuations for plane wave under weak anisotropic non-Kolmogorov turbulence

Following Tatarskii [22], the temporal power spectrum of irradiance fluctuations $W_I(\omega)$ can be defined by the Fourier transform of the temporal covariance function of irradiance fluctuations $C_I(t)$ according to [22]:

$$W_I(\omega)=4\int_0^\infty C_I(t)\cos(\omega t)\,dt. \tag{4}$$