



Robust finite-time global synchronization of chaotic systems with different orders



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ABSTRACT

This article proves that the robust finite-time synchronization behavior can be achieved for the chaotic systems with different orders. Based on the Lyapunov stability theory and using the nonlinear feedback control, sufficient algebraic conditions are derived to compute the linear controller gains. These gains are then used to achieve the robust finite time increasing order and reduced order synchronization of the chaotic systems. This study also discusses the design of a controller that accomplishes the finite time synchronization of two chaotic systems of different dimensions under the effect of both unknown model uncertainties and external disturbances. Numerical simulation results are furnished to validate the theoretical findings.

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1. Introduction

Chaos synchronization is one of the emerging fields in nonlinear sciences. For the last two decades, chaos synchronization has been extensively investigated in depth, particularly after the pioneering work of Pecora and Carroll [1]. Synchronization is immediately interesting for its successful applications in secure communications [2], medicine [3], power electronic systems [4], cryptography [5], laser [6], physical systems [7] and elsewhere. In this direction, chaos synchronization has been accomplished, using several control methods and techniques, such as the nonlinear active control [7], linear feedback control [8], active control [9], sliding mode control [10], nonlinear control [11], adaptive control [12], optimal control [13], and lag synchronization [14]. The main focus of the control laws for chaos synchronization is to design a controller that forces the trajectories of the slave system with initial condition $y_i(0)$ to asymptotically track the trajectories of the master system with initial condition $x_i(0)$ i.e. $\lim_{t \rightarrow \infty} (y(t) - x(t)) = 0$ [15]. Most of the research in this area has addressed chaotic synchronization of the same order systems. Recently, the research trend has been diverted to study the chaos synchronization of systems having different orders [16–21]. The different order chaotic systems synchronizations are fundamentally of two types. These types are named as, increasing order synchronization (IOS) and the reduced-order synchronization (ROS). In the IOS/ROS, the dimension of the master system is smaller/greater than that of the slave system. The ROS of the Rossler system and Duffing equation with the estimation of uncertain parameters has been investigated in [17–19]. The backstepping

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method [18] is used to synchronize the Rossler system and Duffing equation. Using the linear errors of corresponding variables and parameters, [20] investigated the generalized synchronizations between uncertain hyperchaotic and uncertain chaotic systems with identification of unknown parameters. [21] studied the IOS and increased-order anti-synchronization between chaotic systems. The controller functions and the parameter update laws are derived based on an adaptive control method for the uncertain parameters. Backstepping design method [22] is applied to achieve reduced-order projective hybrid combination synchronization and reduced-order hybrid combination synchronization of three chaotic systems. On the other hand, most of the finite time synchronization control schemes [23–25] are developed for the synchronization of chaotic systems with the same orders. Recently, based on the finite-time stability theory, [26] has discussed the finite-time generalized synchronization of chaotic systems with different order.

However, following list describes the limitations of the studies mentioned above;

- (i) The synchronization error does not converge in a finite time [16–22].
- (ii) The nonlinear terms in these control laws have been cancelled [16–22,26]. This cancelation offers difficulties due to the presence of uncertainties in the system. These types of cancellations are a special form of feedback linearization. The feedback linearization based controllers are known for computing large control signals [27]. These large control signals are responsible for the inefficient use of energy.
- (iii) These control methods [16–22,26] are not robust with respect to the unknown model uncertainties and external disturbances. This property gives rise to the loss of the original information in the receiver end.

Synchronization of chaotic systems with different order can be found in many natural and artificial systems such as in the human brain, chaotic laser communications and synchronization in the cells of paddlefish [16] etc. For example, in the case of thalamic neurons, which is reasonable if their order is different from one of the hippocampal neurons [17]. Another synchronization example occurs between the heart and lungs. Synchronization between chaotic systems with different orders is significant in the information processing, in physical phenomena, social and biological systems due to complexities in their chaotic attractors [19,20]. Topological properties of the two chaotic systems and the traces change in their respective trajectories with time are different. These features increase the security in the communications channels. The finite time controls have demonstrated better robustness and disturbance rejection properties. The finite time ROS/IOS of chaotic systems can be used for data compression purpose. This compression enhances the data security in the transmission process. Basically, reducing/increasing the order dimensions in the data transmission process can be reconstructed at the receiver end to the original [28].

Motivated by the aforementioned, in this paper, the design of the robust finite time control laws has been proposed for the ROS and IOS. The Lyapunov theory [27] is used to prove the finite time convergence of the synchronization errors in the presence of system's uncertainty and the disturbances. These control laws do not cancel the nonlinear terms appeared in the error dynamics. To the best of our knowledge, the proposed controller has not been investigated in the literature. Two examples are discussed to illustrate the performance of the current study, (i) the finite time IOS between the Lorenz chaotic [29] and the Lorenz type hyperchaotic [30] systems, (ii) the finite time ROS between the Lorenz type hyperchaotic and the Lorenz chaotic systems. The main contributions of this work include, (a) In constructing the feedback controller, the nonlinear functions are not eliminated, that improves the performance of the proposed controller in terms of increased transient speed and smoothness tracking signals of the closed-loop, (b) The Lyapunov functions are constructed according to the needs of the system which can be adjusted according to the requirements. Furthermore, we take a theoretical approach to the problem in hand and suggest some future work. Numerical simulation results show that the designed finite time synchronization control strategy is effective, reliable and convenient to implement for synchronization of a class of chaotic systems with different orders. The rest of the paper is organized as follows:

Section 2 presents the problem statement and a theory for the proposed finite time IOS and ROS schemes. In Section 3, descriptions of the chaotic Lorenz system and Lorenz type hyperchaotic systems are discussed and solved the IOS and ROS problems. This paper is concluded in Section 4.

2. Problem statement and theory for the proposed finite time IOS and ROS schemes

In this section of the paper, we briefly describe a theory for the proposed finite time IOS and ROS schemes. Consider the following lemmas and preliminaries,

2.1. Theory for the proposed finite time IOS

2.1.1. Problem statement

Let us consider a master chaotic system that is defined as follows:

$$\dot{X}(t) = A_1(X(t)) + F_1(X(t)) + D(X(t)) \quad (1)$$

where, $X(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in R^n$ is the state vector, $A_1 \in R^{n \times n}$ is the constant coefficient matrix which consists of the parameters of the system, $F_1 \in R^{n \times 1}$ is the nonlinear continuous differentiable vector function without the parameter

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