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Research of band gap properties based on two-dimensional photonic crystal with mixed shapes of rods

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ABSTRACT

Two new structures of photonic crystals were designed. The band gap properties of photonic crystals with square and circular dielectric rods mixed arrangement are researched. The band gap properties of mixed shapes rods photonic crystal are calculated and compared with the crystals with square rods or round rods by using plane wave expansion method. Simulation results show that for the square lattice, mixed shapes of rods make the higher-order bands of TM modes moving toward the low frequency range. The gap bands' widths and locations are between the parameters of square and round rods photonic crystal. In triangle lattice, a significant band gap is presented in photonic crystal with mixed shapes of rods in TE mode, while it is almost not presented in square and round rods crystals. The phenomenon of bands moving toward the low frequency range is also found in the triangle lattice mixed shapes rods photonic crystal. The reasons of the results in the vision were analyzed.

1. Introduction

A photonic crystal [1] is a periodic arrangement of media with differing dielectric constants. The photonic crystal also has energy band structure just like semiconductor crystal. When the lattice and dielectric constants are designed appropriately, there may be gaps in the energy band structure that electrons cannot transmit with certain frequencies. This is one of the most important characteristics of photonic crystal–photonic band gaps [2].

There are no transmission modes (or photon state density is zero) in photonic band gaps, meaning that electromagnetic wave is forbidden to propagate with certain frequencies. Generally, the wider the band gap of photonic crystal is, the more stable performance will be, and it will have higher application value. There are many factors have an effect on the widths of band gaps. Previous studies on photonic band gaps used rods with the same shape. They got the laws of factors' influence on properties of photonic band gaps by changed the shape of lattice, fill factor and the ratio of medias' dielectric constants to regulate band gaps' widths and positions [3,4]. So finding a structure which has better performance is a great aim for researchers. The 3-dimensional (3D) photonic crystal is hard to manufacture. At the same time, the manufacture of 2-dimensional (2D) photonic crystal is easier than 3D and also has photonic band gaps.

In this paper, new structures with different shapes of rods mixed arrangement based on 2D photonic crystal are designed. Using different shapes of dielectric rods mixed arrangement is a new method to adjust the band gaps' properties. Their photonic band gaps are compared with same shape rods photonic crystals' by plane wave expansion method. The results show differences between them.

2. Building simulation models

In 2D situation, there may be gaps in the energy band structure of the crystal, meaning that electromagnetic wave is forbidden to propagate with certain directions. If in certain frequencies, the gap can extend to cover all possible propagation directions, resulting in a complete band gap. Different structures make different photonic band gaps. The simulation models are shown in Fig. 1, in which mixed shapes of rods photonic crystals mean the crystal structures with different shapes of rods mixed arrangement.

The simulation models of photonic band gaps are given by next equations.

The propagation of electromagnetic wave in a photonic crystal is governed by the four Maxwell equations:

$$\nabla \times \left[\frac{1}{\varepsilon(\boldsymbol{x}_{12})}\nabla \times H\right] = \frac{\omega^2}{c^2}H \quad (1)$$

$$\nabla \times \left[\frac{1}{\varepsilon(\boldsymbol{x}_{12})}\nabla \times E\right] = \frac{\omega^2}{c^2}E \quad (2)$$

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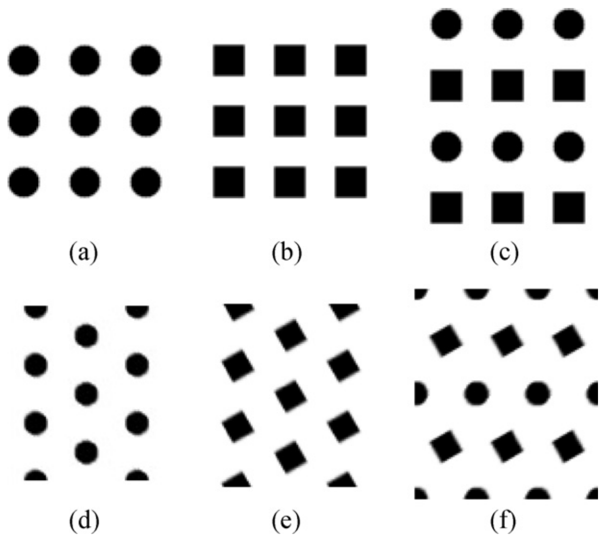

Fig. 1. Structures of photonic crystals. (a) Round rods in square lattice; (b) square rods in square lattice; (c) square and round rods mixed arrangement in square lattice; (d) round rods in triangle lattice; (e) square rods in triangle lattice; (f) square and round rods mixed arrangement in 'triangle lattice.

The variable $\boldsymbol{x}_{12}$ is the plane which the 2D photonic crystal structure in. We can formulate the problem in terms of H first by plane wave expansion method [5]:

$$H(\boldsymbol{x}_{12}, t) = H_0(\boldsymbol{x}_{12}, \omega)e^{-i\omega t} = (0, 0, H_3(\boldsymbol{x}_{12}, \omega))e^{-i\omega t} \quad (3)$$

$$E(\boldsymbol{x}_{12}, t) = E_0(\boldsymbol{x}_{12}, \omega)e^{-i\omega t} = (E_1(\boldsymbol{x}_{12}, \omega), E_2(\boldsymbol{x}_{12}, \omega), 0)e^{-i\omega t} \quad (4)$$

Fourier series expansions of $\varepsilon^{-1}(\boldsymbol{x}_{12})$, $H_3(\boldsymbol{x}_{12},\omega)$ are:

$$\varepsilon^{-1}(\boldsymbol{x}_{12}) = \sum_{\boldsymbol{G}} \boldsymbol{K}(\boldsymbol{G})e^{iG\cdot\boldsymbol{x}_{12}} \quad (5)$$

$$H_3(\boldsymbol{x}_{12}, \omega) = \sum_{\boldsymbol{G}} A(\boldsymbol{K}+\boldsymbol{G})e^{i(\boldsymbol{K}+\boldsymbol{G})\cdot\boldsymbol{x}_{12}} \quad (6)$$

$\boldsymbol{K}$ is the Fourier coefficient of $\varepsilon^{-1}(\boldsymbol{x}_{12})$, $\boldsymbol{G} = h_1\boldsymbol{b}_1 + h_2\boldsymbol{b}_2$ is the 2D reciprocal lattice vector, h_1, h_2 are integers.

The reciprocal lattice vector of photonic crystal with square lattice:

$$\boldsymbol{b}_1 = \frac{2\pi}{a}\boldsymbol{x}_1, \quad \boldsymbol{b}_2 = \frac{2\pi}{a}\boldsymbol{x}_2 \quad (7)$$

The reciprocal lattice vector of photonic crystal with triangle lattice:

$$\boldsymbol{b}_1 = \frac{2\pi}{a}\left(\boldsymbol{x}_1 - \frac{\sqrt{3}}{3}\boldsymbol{x}_2\right), \quad \boldsymbol{b}_2 = \frac{2\pi}{a}\left(-\frac{2\sqrt{3}}{3}\boldsymbol{x}_2\right) \quad (8)$$

Derivation of Eq. (1), using (3) and (4):

$$\sum_{\boldsymbol{G}'} \boldsymbol{K}(\boldsymbol{G}-\boldsymbol{G}')(\boldsymbol{k}+\boldsymbol{G}')(\boldsymbol{k}+\boldsymbol{G})A(\boldsymbol{k}+\boldsymbol{G}') = \frac{\omega^2}{c^2}A(\boldsymbol{k}+\boldsymbol{G}) \quad (9)$$

$\boldsymbol{k}$ is the wave vector in first Brillouin zone. For electric fields, we can also get

$$\sum_{\boldsymbol{G}} \boldsymbol{K}(\boldsymbol{G}-\boldsymbol{G}')\left|\boldsymbol{k}+\boldsymbol{G}\right|\left|\boldsymbol{k}+\boldsymbol{G}'\right|C(\boldsymbol{k}+\boldsymbol{G}') = \frac{\omega^2}{c^2}C(\boldsymbol{k}+\boldsymbol{G}) \quad (10)$$

Because $\boldsymbol{K}$ is the Fourier coefficient of $\varepsilon^{-1}(\boldsymbol{x}_{12})$, it is given by

$$\varepsilon(\boldsymbol{G}) = \frac{1}{S}\int_S \varepsilon(\boldsymbol{x}_{12})e^{-i\boldsymbol{G}\cdot\boldsymbol{x}_{12}}\,d\boldsymbol{x}_{12} \quad (11)$$

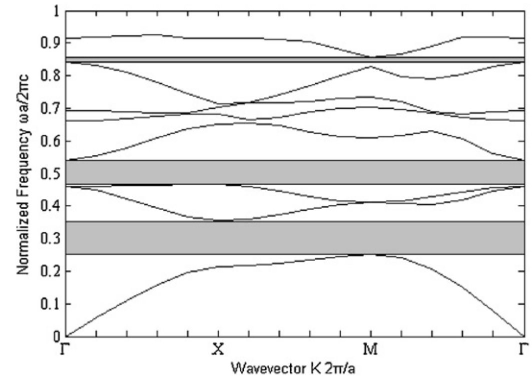

Fig. 2. Gap map of TM mode for photonic crystal with round rods in square lattice.

In which S is the area of primitive cell. Its integral equation is:

$$\varepsilon(\boldsymbol{G}) = \begin{cases} \varepsilon_b + f(\varepsilon_a - \varepsilon_b) & \boldsymbol{G} = 0 \\ (\varepsilon_a - \varepsilon_b)I(\boldsymbol{G}) & \boldsymbol{G} \neq 0 \end{cases} \quad (12)$$

$f = S_{x_{12}}/S$ is fill factor. When shapes of rods changed, f changed with it. $I(\boldsymbol{G})$ is given by

$$I(\boldsymbol{G}) = \frac{1}{S}\int_{S_{\boldsymbol{x}_{12}}} e^{-i\boldsymbol{G}\cdot\boldsymbol{x}_{12}}\,d\boldsymbol{x}_{12} \quad (13)$$

The variable $S_{\boldsymbol{x}_{12}}$ is the cross section area of rods. For round rods

$$I(\boldsymbol{G}) = 2f\frac{J_1(\boldsymbol{G}r)}{\boldsymbol{G}r} \quad (14)$$

J_1 is first order Bessel function. The variable r is radius of round rods. For square rods

$$I(\boldsymbol{G}) = fF(G_1, l)F(G_2, l) \quad (15)$$

In which l is the side length of the square rod. $F(K,X)$ is defined as

$$F(K, X) = \begin{cases} 1 & K = 0 \\ \dfrac{\sin(KX/2)}{KX/2} & K \neq 0 \end{cases} \quad (16)$$

3. Result and analysis

In this paper, two kinds of photonic crystals structures with different lattice shape are computed, one is square lattice [6] and the other is triangle lattice. For each lattice shape, there are round rods, square rods and round and square rods mixed arrangement photonic crystals. We set the background media as air and rods dielectric constant ε to 12.

3.1. Photonic crystal with square lattice

Looking into photonic crystal with square lattice, we assume that the ratio of rods radius and lattice constant is 0.25, the sides of square rods parallel to the primitive reciprocal lattice vectors.

The gap map of TM mode for photonic crystal with round rods in square lattice is plotted in Fig. 2.

We can see 3 band gaps in Fig. 2. Their relative widths are 34.34%, 14.54%, 1.81%, center normalized frequencies are 0.302, 0.503, 0.8481.

Fig. 3 shows the band structures of photonic crystal with square rods in square lattice. There are also 3 gaps which relative widths are 27.48%, 7.94%, 2.25% and center normalized frequencies are 0.2735, 0.4635, 0.6694. The gaps of round rods photonic crystal are found that have the relative widths reduction and the bands' highest normalized frequency is lower than round rods crystal.