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Optimal Kalman Filter for state estimation of a quadrotor UAV

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ABSTRACT

In this work, the main objective is to study the Optimal Kalman Filtering (OKF) method for estimating the state vector of a small quadrotor UAV through incorporating the internal disturbances including the white Gaussian process and measurement noises. Firstly, the kinematic and dynamic model of the quadrotor is transformed into a discrete-time system via the linear extrapolation method. Secondly, for the sake of performing the high accuracy position and attitude tracking control of the quadrotor, the discrete-time flight controller is designed using second order discrete-time sliding mode technique. In addition, the estimation of the quadrotor aircraft's state vector is carried out with the use of OKF. The performance of the combination between the flight controller and the OKF is evaluated through simulation tests. Extensive simulation results show that the combined strategy has a good performance in terms of variance and state estimation.

1. Introduction

In recent years, the aerial robotics has been a fast-growing of robotics and multirotor aircraft, especially unmanned aerial vehicles (UAVs), such as the quadrotor (Fig. 1). Low cost and small-size flying platforms are becoming broadly available, and some of these platforms are able to lift relatively high payloads and provide an increasingly broad set of basic functionalities [1]. Quadrotor UAVs already have sufficient payload and flight endurance to support a number of indoor and outdoor applications, and the improvements of battery and other technology are rapidly increasing the scope for commercial opportunities [2]. However, as a necessity for a successful mission, the quadrotor design must be excellent in point of view of the navigation and control system because of the absence of the pilot who can take initiative. The navigation and control system design of the autonomous aircraft is an important topic that is under discussion [3,4].

The necessary state variables of an autonomous quadrotor UAV were estimated via the Kalman Filter (KF) under the condition of sufficient measurements [5]. It is essential that these state variables including position and attitude are accurately obtained to control the aircraft successfully. Meanwhile, both the actuators and sensors should be kept fault free for such an accurate state estimation. Otherwise the KF gives inaccurate results and even derives by time. Therefore the filter should be built robust in order to achieve fault tolerance in the design of the quadrotor autonomous navigation and control system.

In this work, the Optimal Kalman Filter (OKF) is used to estimate the state variables in the presence of the white Gaussian process and measurement noises, which are caused by the actuator and sensor faults, respectively. The OKF method to the state estimation is quite sensitive to any malfunctions. Moreover, OKF algorithm is adopted here to estimate the state variables from the measurement values of sensors. Considering that the outputs rather than the state variables are obtained by the controller, which holds more substantial reality than [6,7] in which the state variables are utilized by the controller directly.

The results relating to the previously mentioned filter have been given recently in [8–12]. Nowadays, many extended filters based on the standard KF have been developed and applied to the solution of specifical problems [4,13–17]. In [13], the extended Kalman Filter (EKF) in its continuous form, the unscented Kalman Filter (UKF) and the spherical simplex unscented Kalman Filter was considered in multibody models. In [4], a robust adaptive Kalman Filter was introduced, which could incorporate measurement and process noise covariance adaptation procedures (R and Q adaptation respectively) and utilize adaptive factors in order to adapt itself against sensor/actuator faults. A novel multiple model estimator for the satellite attitude determination system composed of gyroscopes and star sensors was proposed in [14], where the gyroscope drift was obtained from a standard Kalman Filter. A novel quaternion estimator called square-root quaternion cubature Kalman Filter was proposed for spacecraft attitude estimation in [15]. In

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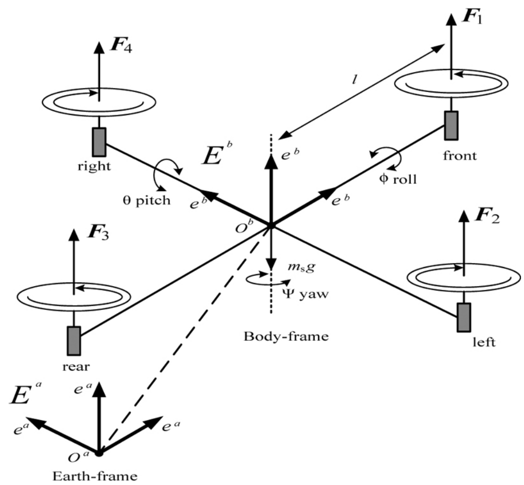

Fig. 1. Quadrotor aircraft.

[16], a resampling ensemble Kalman Filter method was introduced for general nonlinear case. Nonlinear Kalman Filtering and Particle Filtering methods for estimating the state vector of UAVs through the fusion of sensor measurements was studied and compared in [17]. It is noted that all the above papers are not related to the design method of controller and stability analysis of closed-loop system.

In this work, the second order discrete-time sliding mode technique is utilized to design a flight controller for the quadrotor. The remind structure of this paper is organized as follows. In Section 2, the kinematic and dynamic model of the quadrotor is given, and the discrete-time flight controller design is introduced. In Section 3, the OKF for the quadrotor state estimation is illustrated. In Section 4, the simulation tests are performed to evaluate the performance of the combination between the flight controller and the OKF. Section 5 just gives a brief summary of the obtained results and concludes the work.

2. Full state feedback control for quadrotor

The kinematic and dynamic model of the quadrotor, shown in Fig. 1, is described in details by Xiong et al. [18,19].

The dynamic characteristics of the quadrotor must be known to build the OKF for the state estimation. In general, six rigid body equations, which consist of three force and three moment equations, are obtained for the quadrotor. According to these papers [18–22], these force and moment equations are written by

$$\begin{cases} \ddot{x} = (\cos\phi\sin\theta\cos\psi + \sin\phi\sin\psi)\dfrac{u_1}{m} - \dfrac{K_1\dot{x}}{m} \\ \ddot{y} = (\cos\phi\sin\theta\sin\psi - \sin\phi\cos\psi)\dfrac{u_1}{m} - \dfrac{K_2\dot{y}}{m} \\ \ddot{z} = (\cos\phi\cos\theta)\dfrac{u_1}{m} - g - \dfrac{K_3\dot{z}}{m} \end{cases} \tag{1}$$

$$\begin{cases} \ddot{\phi} = \dot{\theta}\dot{\psi}\dfrac{I_y - I_z}{I_x} + \dfrac{J_r}{I_x}\dot{\theta}\Omega_r + \dfrac{l}{I_x}u_2 - \dfrac{K_4 l}{I_x}\dot{\phi} \\ \ddot{\theta} = \dot{\phi}\dot{\psi}\dfrac{I_z - I_x}{I_y} - \dfrac{J_r}{I_y}\dot{\phi}\Omega_r + \dfrac{l}{I_y}u_3 - \dfrac{K_5 l}{I_y}\dot{\theta} \\ \ddot{\psi} = \dot{\phi}\dot{\theta}\dfrac{I_x - I_y}{I_z} + \dfrac{1}{I_z}u_4 - \dfrac{K_6}{I_z}\dot{\psi} \end{cases} \tag{2}$$

where x, y and z are the coordinates of the quadrotor center of mass in the inertial frame. I_x, I_y and I_z are the moments of inertia among x, y and z directions, respectively. ϕ, θ and ψ are the roll, pitch and yaw Euler angles. K_i $(i = 1, 2, 3, 4, 5, 6)$ is drag coefficients and positive constants. J_r is the moment of inertia for each motor. $\Omega_r = \Omega_1 - \Omega_2 + \Omega_3 - \Omega_4$, Ω_i $(i = 1, 2, 3, 4)$ is the ith propeller speed, Ω_r is the overall speed of propellers. m is the total mass of the aircraft. g is the acceleration of gravity. l is the distance from the center of each rotor to the center of gravity. $F_i = b\Omega_i^2$ $(i = 1, 2, 3, 4)$ is the thrust generated by the ith rotor.

$$\begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} = \begin{bmatrix} b & b & b & b \\ 0 & -bl & 0 & bl \\ -bl & 0 & bl & 0 \\ d & -d & d & -d \end{bmatrix} \begin{bmatrix} \Omega_1^2 \\ \Omega_2^2 \\ \Omega_3^2 \\ \Omega_4^2 \end{bmatrix} \tag{3}$$

where b is thrust coefficient, d is drag coefficient.

The designed controllers are given as follows [19].

$$\begin{aligned} u_1 &= \frac{m[c_z(\dot{z}_d - \dot{z}) + \ddot{z}_d + g + \varepsilon_1 sgn(s_1) + \eta_1 s_1] + d_1}{\cos\phi\cos\theta} \\ u_4 &= I_z[c_\psi(\dot{\psi}_d - \dot{\psi}) + \ddot{\psi}_d + \varepsilon_2 sgn(s_2) + \eta_2 s_2] + d_2 \\ u_3 &= \frac{I_y}{lc_3}\left\{ \begin{matrix} c_1(\ddot{x}_d - \ddot{x}) + c_2(\dot{x}_d - \dot{x}) + c_3\ddot{\theta}_d \\ +c_4(\dot{\theta}_d - \dot{\theta}) + \varepsilon_3 sgn(s_3) + \eta_3 s_3 \end{matrix} \right\} + d_3 \\ u_2 &= \frac{I_x}{lc_7}\left\{ \begin{matrix} c_5(\ddot{y}_d - \ddot{y}) + c_6(\dot{y}_d - \dot{y}) + c_7\ddot{\phi}_d \\ +c_8(\dot{\phi}_d - \dot{\phi}) + \varepsilon_4 sgn(s_4) + \eta_4 s_4 \end{matrix} \right\} + d_4 \end{aligned} \tag{4}$$

where the switching sliding surfaces are

$$\begin{aligned} s_1 &= c_z(z_d - z) + (\dot{z}_d - \dot{z}) \\ s_2 &= c_\psi(\psi_d - \psi) + (\dot{\psi}_d - \dot{\psi}) \\ s_3 &= c_1(\dot{x}_d - \dot{x}) + c_2(x_d - x) + c_3(\dot{\theta}_d - \dot{\theta}) + c_4(\theta_d - \theta) \\ s_4 &= c_5(\dot{y}_d - \dot{y}) + c_6(y_d - y) + c_7(\dot{\phi}_d - \dot{\phi}) + c_8(\phi_d - \phi) \end{aligned}$$

where the state variables with the subscript d are reference values, c_z, c_ψ and c_i $(i = 1, \ldots, 8)$ are the coefficients of the defined switching sliding surfaces, besides, c_z and c_ψ are positive constants, c_i $(i = 1, \ldots, 8)$ are obtained by Hurwitz [19], d_i $(i = 1, 2, 3, 4)$ are the disturbances as $d_1 = K_3\dot{z}$, $d_2 = -\dot{\phi}\dot{\theta}(I_x - I_y) + K_6\dot{\psi}$, $d_3 = [-\dot{\phi}\dot{\psi}(I_z - I_x) + J_r\dot{\phi}\Omega_r$, $d_4 = [-\dot{\theta}\dot{\psi}(I_y - I_x) - J_r\dot{\theta}\Omega_r + K_4 l\dot{\phi}]/l$. The coefficients ε_i and η_i $(i = 1, 2, 3, 4)$ are positive.

Considering that the force and moment equations of the kinematic and dynamic model of the quadrotor aircraft are the second order equations, and, in order to estimate the state vector for navigation and tracking control, a second order discrete-time system is considered via linear discretization in the following state-space form

$$\begin{aligned} x_1(k+1) &= x_1(k) + x_2(k)T \\ x_2(k+1) &= x_2(k) + f(k)u(k)T + d(k)T \end{aligned} \tag{5}$$

here, $x_1(k)$ and $x_2(k)$ are the state variables, T is a constant sampling period, $f(k)$ is the coefficient term of the control input, $u(k)$ is the control input of the system, $d(k)$ is taken as the disturbance term.

3. Optimal Kalman Filter (OKF)

In order to enable navigation of the quadrotor when estimating its state vector by fusing measurements from on-board sensors, the Optimal Kalman Filtering is applied. In the discrete-time case, a linear dynamical system is assumed to be expressed in the form of a discrete-time state model [17,23,24].

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) + Gw(k) \\ y(k) &= Cx(k) + v(k) \end{aligned} \tag{6}$$