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Mathematical models of light waves in Brillouin-scattering fiber-optic gyroscope resonator

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ABSTRACT

The Brillouin-scattering fiber-optic gyroscope (BFOG) is a new-generation miniaturized high-precision fiber-optic gyroscope. By focusing on the light characteristics of the BFOG resonator, we derive a recursive relationship for the intensity of light of different orders by using the coupled equations for stimulated Brillouin scattering. We then establish mathematical models of the light characteristics, threshold power analysis, and optimum resonance condition analysis. Based on these models, we derive a functional relationship between the incident pump light intensity, light intensity in the cavity, the intensity of each Stokes wave order, and emitted light intensity. Finally, we simulate and quantify the threshold power and resonance critical condition influenced by several main factors including the insertion loss, coupling coefficient, and transmission loss.

1. Introduction

The Brillouin-scattering fiber-optic gyroscope (BFOG) is a precise instrument for sensing the optical Sagnac effect using a fiber resonator. The BFOG detects beat frequency signals from Brillouin scattering that are proportional to the angular rate of the carrier. A core component of the BFOG is the fiber resonator, in which pump and Stokes light of different orders propagates simultaneously [1,2]. However, only the first-order Stokes light wave is suitable for the gyroscope beat signal detection; other orders and waves affect the stability of the beat signal [3] and thus the performance of the BFOG. The light waves that exist in a resonator are complex, and it is important to study the features of these waves and investigate the coupled equations of the pump and Stokes light when stimulated Brillouin scattering occurs [4,6]. Here, we develop a mathematical model of the light characteristics in the resonator of a BFOG to describe how the first- and higher-order stimulated Brillouin-scattering light wave intensity changes. We discuss the relationship between the pump and Stokes light and also provide a theoretical basis for setting the BFOG resonant light-cycle operating point to ensure that only first-order Stokes light is present in the optical return path.

2. Light wave characteristics in the BFOG

The BFOG (Fig. 1) consists of a high-power narrow-spectrum laser source, fiber resonator, and signal processing unit [8,11]. Pump light from the laser enters the fiber resonator through the fiber coupler, and an increase in the optical power induces first-order Stokes light waves, B_1 and B_2, that propagate in a direction opposite to that of the pump light P_1 and P_2. Both the Sagnac effect in the BFOG and changes to the resonant frequency will produce a difference in the resonance frequencies of B_1 and B_2 [9,10], which results in interference, and the interference signal is processed by the detection circuit. As the power of the pump light exceeds the Brillouin scattering threshold power, an increase in the pump power will result in an increase in the power of the first-order Stokes light [5,7] and stimulate the production of second- and higher-order Stokes light waves.

The Brillouin scattering threshold intensity $I_{c,\text{th}}$ in the fiber resonator for exciting the first-order Stokes light wave is given by

$$I_{c,\text{th}} = -\frac{\ln R^2}{g_B L} = \frac{2A_{\text{eff}}}{g_B L}(\gamma + \alpha L) \tag{1}$$

$$R = \sqrt{1-k}\sqrt{1-\gamma}\exp(-\alpha L/2) \tag{2}$$

where R is the amplitude transmission coefficient, g_B (=2.098 × 10^{-11} m/W) is the Brillouin gain coefficient, A_{eff} (=60 μm^2) is the effective cross-sectional area of the fiber, γ is the insertion loss of

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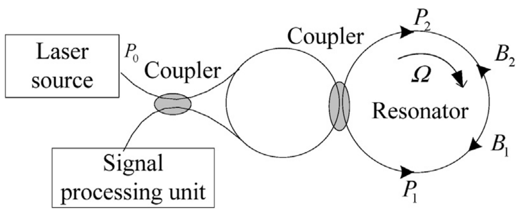

Fig. 1. Basic structure of the BFOG.

the optical fiber coupler, α is the transmission loss coefficient of the fiber ring, L is the resonator length, and k is the intensity coupling coefficient of the coupler.

The threshold intensity $I_{p,\mathrm{th}}$ of the laser source for exciting the first-order Stokes light wave is

$$I_{p,\mathrm{th}} = \frac{(\ln R^2)^2}{4(1-\gamma)k} I_{c,\mathrm{th}} = \frac{2A_{\mathrm{eff}}}{g_B L}\frac{(\gamma+\alpha L)^2}{1-\gamma} \tag{3}$$

Before the first-order Stokes light wave is excited, the average intensity $\bar{I}_{P1}$ of the pump light wave P_1 is

$$\bar{I}_{P1}/I_{c,\mathrm{th}} \approx I_p^{\mathrm{in}}/I_{p,\mathrm{th}} \tag{4}$$

where I_p^{in} is the intensity of incident pump light. Thus, $\bar{I}_{P1}$ in the resonator increase linearly with an increase in the power of the laser source before the first-order Stokes light wave is excited.

After the first-order Stokes light wave is excited, $I_{p,\mathrm{th}}$ remains constant with an increase in the pump intensity, and the energy is instead converted into the intensity of the first-order Stokes light $\bar{I}_{B1}$ and loss in the fiber resonator:

$$\bar{I}_{B1}/I_{c,\mathrm{th}} = \sqrt{I_p^{\mathrm{in}}/I_{p,\mathrm{th}}} - 1 \tag{5}$$

when I_p^{in} increases to $4\times I_{p,\mathrm{th}}$, the second-order Stokes light wave B_{1_2} is excited, from which we can derive

$$\bar{I}_{P1}/I_{c,\mathrm{th}} = I_p^{\mathrm{in}}/(4\times I_{p,\mathrm{th}}) \tag{6}$$

$$\bar{I}_{B1\_2}/I_{c,\mathrm{th}} = I_p^{\mathrm{in}}/(4\times I_{p,\mathrm{th}}) - 1 \tag{7}$$

Therefore, an increase in the power of the laser source excites higher-order Stokes light waves in the resonator.

3. Threshold power analysis model

The differential equations of the pump and Stokes light wave distributions along the length of the fiber resonator z are

$$\frac{dI_s(z)}{dz} = -g_B I_P(z) I_s(z) + \alpha I_s(z) \tag{8}$$

$$\frac{dI_P(z)}{dz} = -g_B I_P(z) I_s(z) - \alpha I_P(z) \tag{9}$$

where $I_s(z)$ and $I_P(z)$ are the intensities of the Stokes and pump light waves, respectively.

Integration of Eq. (9) yields

$$I_P(z) = I_P(0)e^{-\alpha z} \tag{10}$$

and from Eqs. (8) and (10), the intensity of the Stokes light wave is given by

$$I_s(0) = I_s(L)e^{g_B I_P(0) L_{\mathrm{eff}} - \alpha L} \tag{11}$$

$$L_{\mathrm{eff}} = \frac{1}{\alpha}(1-e^{-\alpha L}) \tag{12}$$

Thus, the threshold power P_{th} needed to excite the first-order Stokes light wave is

$$P_{\mathrm{th}} = G\frac{K_B A_{\mathrm{eff}}}{g_B L_{\mathrm{eff}}} \tag{13}$$

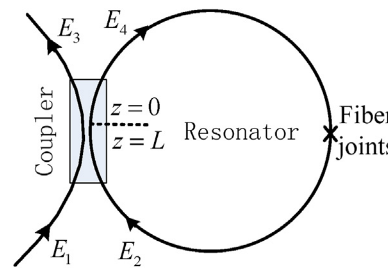

Fig. 2. Schematic diagram of the fiber resonator.

where K_B is the polarization factor determined by the degree of polarization, L_{eff} is the effective length of the fiber, and G is the total Brillouin gain coefficient.

Eq. (13) can also be rewritten as

$$P_{\mathrm{th}} = G\frac{K_B A_{\mathrm{eff}}\alpha}{g_B(1-e^{-\alpha L})} \tag{14}$$

which shows that the threshold power is affected by the fiber length and fiber loss. A longer fiber with a lower loss will thus result in a lower threshold power.

4. Optimum resonance condition analysis model

The stability of the beat signal output from the resonator directly affects the performance of the BFOG [12], and so it is essential to determine the optimum resonance condition.

A schematic diagram of the fiber resonator is shown in Fig. 2. The light distribution field is divided into four components, E_1, E_2, E_3, and E_4, that are described by the following functions:

$$\left|E_3\right|^2 + \left|E_4\right|^2 = (1-\gamma)\left(\left|E_1\right|^2 + \left|E_2\right|^2\right) \tag{15}$$

$$E_2 = E_4\cdot(1-\alpha_0)\exp(-\alpha L + j\beta L) \tag{16}$$

$$E_3 = E_1\cdot\frac{\sqrt{1-\gamma}}{\sqrt{1-k}}\left(1-\frac{k}{1-\sqrt{(1-k)(1-\alpha_0)(1-\gamma)e^{-2\alpha L}}e^{j\beta L}}\right) \tag{17}$$

$$E_4 = E_1\cdot\frac{j\sqrt{1-\gamma}\sqrt{k}}{\sqrt{1-k}-\sqrt{(1-k)(1-\alpha_0)(1-\gamma)e^{-2\alpha L}}e^{j\beta L}} \tag{18}$$

where βL is the phase of the light wave propagating in the fiber loop, α_0 is optical fiber splice loss in resonant, β is the light propagation constant, and j is the imaginary unit.

Under optimum resonance conditions, the energy of the resonator coupler exit port is zero, which defines

$$1-\frac{k}{1-\sqrt{(1-k)(1-\alpha_0)(1-\gamma)e^{-2\alpha L}}e^{j\beta L}} = 0 \tag{19}$$

Setting $M = (1-k)(1-\alpha_0)(1-\gamma)e^{-2\alpha L}$, Eq. (19) can be simplified as

$$1-k-\sqrt{M}\cos(\beta L) - j\sqrt{M}\sin(\beta L) = 0 \tag{20}$$

The resonance phase angle condition can be determined by setting the imaginary part to zero:

$$\beta L = q\cdot 2\pi \tag{21}$$

where q is an integer. In a similar manner, the resonance amplitude condition is given by setting the real part to zero:

$$k = k_r = 1-(1-\gamma)(1-\alpha_0)e^{-2\alpha L} \tag{22}$$