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Adaptive estimation for delayed neural networks with Markovian jumping parameters

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ABSTRACT

In this letter, the adaptive estimation problem of the neuron states is studied for delayed neural networks with Markovian jumping parameters. Through employing a new nonnegative function and the *M*-matrix method, some sufficient conditions are derived to ensure the almost sure asymptotical stability for the dynamics of estimator error. Several suitable parameters update laws are found by the adaptive feedback control technique. Finally, a simulation example is provided to illustrate the effectiveness of the proposed adaptive estimation method.

1. Introduction

In reality, many neural networks may undergo abrupt changes in their parameters and structure caused by some phenomena such as component failures and sudden environmental disturbances etc. In this case, there exist finite modes, and the modes may be jumped from one to another at different times. And the dynamic behavior of neural networks contains inherent time delays, which may cause instability or oscillation. This kind of neural networks are widely studied by many scholars [1–8].

Meanwhile, some state variables are unknown and needed to be estimated in the practical neural networks. By a set of measured system quantities, the goal of state estimation is to find an estimation of system quantities. Furthermore, the adaptive control can help to deliver both stability and good response for systems with variability in parameters that can either be predicted or uncertain. The aim of adaptive control is to adjust the unknown parameters.

The adaptive estimation problem has been extensively investigated over the last decade due to their successful applications in many areas (see e.g. [9–14]), such as missile defence system, the Kalman filter, nonlinear systems, etc. In [9], Lyapunov-based adaptive state estimation of continuous-time and discrete-time nonlinear stochastic systems is considered, the adaptive state and parameter estimators with ultimately exponentially bounded estimator errors is presented by using stochastic counterparts of Lyapunov stability theory, and sufficient conditions are given in terms of the solvability of LMIs. In [12], adaptive estimation for linear time varying system are investigated, and the stability of the overall estimation scheme is established by the solvability of LMIs. It should be pointed out that, up to now, the problem of adaptive estimation based on *M*-matrix for delayed neural networks with Markovian jumping parameters has received very little research attention, which is the motivation of this letter.

The main contributions of this letter can be highlighted as follows: (1) A new adaptive estimation for delayed neural networks with Markovian jumping is addressed; (2) Via the adaptive feedback control method, suitable parameters update laws are found; (3) A new design algorithm of the adaptive estimator is provided by a new nonnegative function and the *M*-matrix method.

2. Problem formulation and preliminaries

Consider the following delayed neural network with n neurons:

$$\dot{x}(t) = -C(r(t))x(t) + A(r(t))f(x(t)) + B(r(t))f(x(t-\tau(t))) + D(r(t)), \tag{1}$$

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where $t \geq 0$ is the time variable, $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in \mathbb{R}^n$ is the state vector of the neural networks, $f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \cdots, f_n(x_n(t))]^T \in \mathbb{R}^n$ denotes the activation function, $\tau(t)$ is the transmission delay satisfying that $0 < \tau(t) \leq \bar{\tau}$ and $\dot{\tau}(t) \leq \hat{\tau} < 1$, where $\bar{\tau}, \hat{\tau}$ are constants. For the sake of simplicity, for $t \geq 0$, we mark $r(t) = i$ and $C(r(t)) = C^i$, $A(r(t)) = A^i$, $B(r(t)) = B^i$ and $D(r(t)) = D^i$ respectively. And $C^i = \mathrm{diag}\{c_1^i, c_2^i, \cdots, c_n^i\}$ has positive and unknown entries $c_k^i > 0$, $A^i = (a_{jk}^i)_{n\times n}$ and $B^i = (b_{jk}^i)_{n\times n}$ are the connection weight and the delayed connection weight matrices, respectively, and are both unknown matrices. $D^i = [d_1^i, d_2^i, \cdots, d_n^i]^T \in \mathbb{R}^n$ is the constant vector. $r(t)$ is a continuous-time Markovian process with right continuous trajectories taking values in a finite set $S = \{1, 2, \cdots, N\}$ with transition probabilities given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \pi_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \pi_{ii}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$, $\lim_{\Delta\to 0}(o(\Delta)/\Delta) = 0$, and $\pi_{ij} \geq 0$, for $j \neq i$, is the transition rate from mode i at time t to mode j at time $t+\Delta$ and $\pi_{ii} = -\sum_{j\neq i}\pi_{ij}$.

The network measurements are assumed to satisfy

$$y(t) = x(t) + g(x(t)), \tag{2}$$

where $y(t) \in \mathbb{R}^m$ is the measurement output, $g(x(t))$ is the neuron-dependent nonlinear disturbances on the network output.

The full-order state estimator is of the form

$$\begin{aligned} \dot{\hat{x}}(t) &= -\hat{C}^i\hat{x}(t) + \hat{A}^i f(\hat{x}(t)) + \hat{B}^i f(\hat{x}(t-\tau(t))) + D^i \\ &\quad + K^i[\hat{x}(t) + g(\hat{x}(t)) - y(t)], \end{aligned} \tag{3}$$

where $\hat{x}(t)$ is the state vector of the state estimator (3), $\hat{C}^i = \mathrm{diag}\{\hat{c}_1^i, \hat{c}_2^i, \cdots, \hat{c}_n^i\}$, $\hat{A}^i = (\hat{a}_{jk}^i)_{n\times n}$ and $\hat{B}^i = (\hat{b}_{jk}^i)_{n\times n}$ are the estimators of unknown matrices C^i, A^i and B^i, respectively. $K^i = \mathrm{diag}\{k_1^i, k_2^i, \cdots, k_n^i\}$ is the estimator gain matrix to be designed.

Let the error state be $e(t) = \hat{x}(t) - x(t)$. As a matter of convenience, we denote $e(t-\tau(t)) = e_\tau(t)$, $f(\hat{x}(t)) - f(x(t)) = \varphi(e(t))$ and $g(\hat{x}(t)) - g(x(t)) = \phi(e(t))$. Then it follows from (1), (2) and (3) that

$$\begin{aligned} \dot{e}(t) &= -\tilde{C}^i\hat{x}(t) - C^i e(t) + \tilde{A}^i f(\hat{x}(t)) + A^i\varphi(e(t)) + \tilde{B}^i f(\hat{x}_\tau(t)) \\ &\quad + B^i\varphi(e_\tau(t)) + K^i[e(t) + \phi(e(t))], \end{aligned} \tag{4}$$

where $\tilde{C}^i = \hat{C}^i - C^i$, $\tilde{A}^i = \hat{A}^i - A^i$ and $\tilde{B}^i = \hat{B}^i - B^i$. Denote $\tilde{c}_j^i = \hat{c}_j^i - c_j^i$, $\tilde{a}_{jk}^i = \hat{a}_{jk}^i - a_{jk}^i$ and $\tilde{b}_{jk}^i = \hat{b}_{jk}^i - b_{jk}^i$, then $\tilde{C}^i = \mathrm{diag}\{\tilde{c}_1^i, \tilde{c}_2^i, \cdots, \tilde{c}_n^i\}$, $\tilde{A}^i = (\tilde{a}_{jk}^i)_{n\times n}$ and $\tilde{B}^i = (\tilde{b}_{jk}^i)_{n\times n}$.

Let $e(t;\xi)$ be the trajectory of the error system (4) under the initial condition $e(\theta) = \xi(\theta)$ on $-h \leq \theta \leq 0$ in $L^2_{\mathcal{F}_0}([-h, 0], \mathbb{R}^n)$. It is obvious that the error system (4) admits a trivial solution $e(t;0) = 0$ corresponding to the initial condition $\xi = 0$.

The main purpose of the rest of this letter is to set up a criterion of adaptive estimation for the system (1)-(4) via employing adaptive feedback control and M-matrix method. Next, we firstly introduce some concepts and lemmas which will be used in the proofs of main results.

Assumption 1. The neuron activation functions $f(x(t))$ in (1) and $g(x(t))$ in (2) satisfy the Lipschitz condition. That is, there exist constants $L_1 > 0$ and $L_2 > 0$ such that $|f(u) - f(v)| \leq L_1|u - v|$ and $|g(u) - g(v)| \leq L_2|u - v|$, $\forall u, v \in \mathbb{R}^n$, respectively.

Definition 1. [15] A square matrix $M = (m_{ij})_{n\times n}$ is called a nonsingular M-matrix if M can be expressed in the form of $M = sI_n - G$ with $s > \rho(G)$ while all the elements of G is nonnegative, where I is the identity matrix and $\rho(G)$ is the spectral radius of G.

Lemma 1. [15] *If $M = (m_{ij})_{n\times n} \in \mathbb{R}^{n\times n}$ with $m_{ij} < 0\ (i \neq j)$, then the following statements are equivalent.*

(i) *M is a nonsingular M-matrix.*
(ii) *M is inverse-positive; that is, M^{-1} exists and $M^{-1} > 0$.*
(iii) *M is positive stable. That is to say, the real part of each eigenvalue of M is positive.*

Consider a n-dimensional delayed differential equation (DDE) with Markovian jumping parameters

$$dx(t) = f(t, r(t), x(t), x_\tau(t))dt \tag{5}$$

on $t \in [0, \infty)$ with the initial condition given by $\{x(\theta) : -\bar{\tau} \leq \theta \leq 0\} = \xi(\theta) \in L^p_{\mathcal{L}_0}([-\bar{\tau}, 0]; \mathbb{R}^n)$.

For $V \in C^{2,1}(R_+ \times S \times \mathbb{R}^n; R_+)$, define an operator $\mathcal{L}$ from $R_+ \times S \times \mathbb{R}^n \times \mathbb{R}^n$ to R by

$$\begin{aligned} &\mathcal{L}V(t, i, x(t), x_\tau(t)) \\ &= V_t(t, i, x(t)) + V_x(t, i, x(t))f(t, i, x(t), x_\tau(t)) \\ &+ \sum_{j=1}^{N} \gamma ij V(t, j, x(t)), \end{aligned}$$

where $V_t(t, i, x(t)) = \frac{\partial V(t,i,x(t))}{\partial t}$, $V_x(t, i, x(t)) = (\frac{\partial V(t,i,x(t))}{\partial x_1}, \frac{\partial V(t,i,x(t))}{\partial x_2}, \cdots, \frac{\partial V(t,i,x(t))}{\partial x_n})$.

Lemma 2. [16] *Let $x \in \mathbb{R}^n$ and $y \in \mathbb{R}^n$. Then $x^Ty + y^Tx \leq \epsilon x^Tx + \epsilon^{-1}y^Ty$ for any $\epsilon > 0$.*

Lemma 3. [17] *Let Assumption 1 holds. Assume that there are functions $V \in C^{2,1}(R_+ \times S \times \mathbb{R}^n; R_+)$, $\psi \in L^1(R_+; R_+)$ and $w_1, w_2 \in C(\mathbb{R}^n; R_+)$ such that*

$$\mathcal{L}V(t, i, x, y) \leq \psi(t) - w_1(x) + w_2(y),$$

$$\forall (t, i, x, y) \in R_+ \times S \times \mathbb{R}^n \times \mathbb{R}^n, \tag{6}$$

$$w_1(0) = w_2(0) = 0, w_1(x) > w_2(x) \quad \forall x \neq 0, \tag{7}$$

and

$$\lim_{|x|\to\infty} \inf_{0\leq t<\infty, i\in S} V(t, i, x) = \infty. \tag{8}$$

Then the solution of equation (5) is almost sure asymptotical stable.

3. Main results

In this section, a new criterion of adaptive estimation for the systems (1)–(4) is given by using adaptive feedback control and M-matrix method.

Theorem 1. *Assume that $M := -\mathrm{diag}\{\underbrace{\eta, \eta, \cdots, \eta}_{N}\} - \Gamma$ is a M-matrix, where $\eta = -2\gamma + \alpha + L^2 + \beta$, $\gamma = \min_{i\in S}\min_{1\leq j\leq n} c_j^i$, $\alpha = \max_{i\in S}(\rho(A^i))^2$, $\beta = \max_{i\in S}(\rho(B^i))^2$. Let $m > 0$ and $\vec{m} = (\underbrace{m, m, \cdots, m}_{N})^T$ (In this situation, $[q_1, q_2, \cdots, q_N]^T := M^{-1}\vec{m} \gg 0$, that is to say, all elements of $M^{-1}\vec{m}$ are positive according to Lemma 2.1). Assume also that*

$$L^2\bar{q} < -\left[\eta q_i + \sum_{k=1}^{N}\gamma_{ik}q_k\right], \quad \forall i \in S, \tag{9}$$

where $\bar{q} = \max_{i\in S} q_i$.