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Fresnel diffraction by a square aperture with rough edge

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ABSTRACT

In this paper, we study Fresnel diffraction of a square aperture with rough edge with Gaussian random process. The diffraction intensity formula depending on the random parameters is deduced by use of the scalar diffraction theory. The simulation calculations of the diffraction of the square aperture with different statistic parameters are performed. The diffraction intensity distributions exhibit elaborately the influence of the random parameters of rough edge on the diffraction of a square aperture. The results show that the larger the lateral correlation length and the larger roughness of the rough edge are, the more severe the modulation of the random spots is and the more unsymmetrical the diffraction distribution is. For the same random parameters, the modulation degree of the random spots changes with the propagation distance. Moreover, we also manufacture practically the random apertures with the help of the laser direct writing technology, and measure the diffraction distribution of the square aperture with random edge. The experimental results are consistent with the theoretic ones. These results will be instructively meaningful for the applications of the diffraction of the practical aperture.

1. Introduction

Diffraction of small aperture, as an optical basic problem in diffraction optics, has been attracted wide attention. The proper aperture can realize the control of the light field, and obtain the needed intensity redistribution and the phase shift of the diffraction field [1,2]. As far, the aperture diffraction or transmission has been applied in different fields, such as the scanning optical microscopy [3], the enhanced transmission [1,4], the birth of optical vortex [5], and so on. As we know, the spatial distribution of diffraction field depends on the shape and the size of aperture, so the diffraction of the aperture with different shape, such as square aperture [4], triangular aperture [1,5], circular aperture [6–10] and elliptical aperture [11–13], has been deeply studied. In most studies including our former work, for simplifying theoretical analysis of the aperture diffraction, the edges of aperture are considered to be perfectly regular. However, the practical aperture manufactured practically by any method has the rough edge, and this rough edge can be also observed clearly under the amplification device. Naturally, the rough edge of aperture can influence the diffraction distribution of the small aperture. Therefore, the study about the diffraction by the aperture with rough edge has the practical significance for estimating the diffraction rule and predicting the effect of the aperture.

Early in 1979, George and Morris carried out the theoretic and experimental research about Fresnel diffraction of serration hole, and they described the diffraction by use of the correlation function [14]. In 1989, Beal and George studied Fraunhofer diffraction of serration circular hole and serration circular disc. They obtained expressions of the electric field and the second central moment in Fourier transform plane and discussed the effects of roughness, correlation angle and correlation function on the diffraction [15]. Kim and Grebel performed the study about Fraunhofer diffraction of aperture with sinusoidal boundary and fractal structure, and analyzed the influence of period of sinusoidal boundary and the fractal dimension on the diffraction of the circular and rectangle apertures [16]. Later, Gu and Gan studied Fresnel diffraction by the circular hole with sinusoidal edge illuminated by continuous and pulsed laser [17]. Among these studies, the aperture edges take the simple sinusoidal or serrated shape. However, the random edge of practical aperture is closer to the Gaussian statistical distribution [18].

In this paper, we concentrate on the study about Fresnel diffraction by square aperture with Gaussian random edge, and investigate the influence of random interface parameters on Fresnel diffraction. Theoretically, the square aperture with random edge is obtained through the numerical simulation under the certain random parameters which satisfying the Gaussian statistics. The method is described in detail in Section 2. Where the diffraction

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intensity by the square aperture with different rough edge is analyzed and the diffraction intensity distributions are provided according to the numerical calculations. The calculation results show how the transverse correlation length and the roughness of the rough edge influence the diffraction of square aperture. The concrete content is shown in Section 3. Section 4 presents the experimental operation and the practical measurement results. In the end, the conclusions of this paper are presented.

2. Production of square aperture with random edge and theoretic analysis of its diffraction

For the aperture manufactured practically, the aperture edge is uneven. Suppose the random height distribution of a rough edge $h(r)$, its random process is usually described by two statistical parameters on basis of the statistical theory [19]. One is the height probability distribution and the other is the height correlation function. Here, we mainly study Gaussian random edge with height probability distribution taking on the Gaussian shape, which is a common random model and also closer to the real case. Its height probability distribution can be expressed by,

$$p(h) = \frac{1}{\sqrt{2\pi}w} \exp\left(-\frac{h^2}{2w^2}\right) \quad (1)$$

where w is the root-mean-square roughness of the edge, and it describes the oscillation degree of the edge diverging average height. Customarily, we simply call it as the roughness. The larger the roughness w is, the rougher the edge is. Moreover, in order to reflect the relevance of the heights at different positions, the autocorrelation function of Gaussian random edge R_h can be phenomenological expressed as [18,19],

$$R_h(r + \Delta r) = \left\langle h(r)h(r + \Delta r)\right\rangle = w^2 \exp\left[-\left(\frac{\Delta r}{\xi}\right)^2\right] \quad (2)$$

where $\langle\cdot\rangle$ represents the ensemble average, ξ is the lateral correlation length of the random edge, and Δr means the position space of two heights. From this above equation, we can see that two heights with the position space limited within the region of the lateral correlation length ξ are correlation. When the position space is larger than ξ, R_h decreases rapidly and two heights are uncorrelated. In order to obtain the random edge, we introduce a function $f(u)$, which is defined as,

$$f(u) = \left[\int_{-\infty}^{\infty} R_h(\Delta r)\exp(i2\pi\Delta r u)\,\mathrm{d}u\right]^{1/2} \quad (3)$$

Then, let $f(u)$ multiply a random distribution $\eta(u)$ limited within $[-\sqrt{3}, \sqrt{3}]$ and with zero average value, we next perform Fourier transform. The height distribution can be obtained finally,

$$h(r) = Re\left[\sqrt{2}\int_{-\infty}^{\infty} f(u)\eta(u)\exp(-i2\pi r u)\,\mathrm{d}u\right] \quad (4)$$

where $Re[]$ represents the real part of the complex integration. Insert Eqs. (2) and (3) into Eq. (4), we can obtain the random distribution with certain w and ξ. This method to produce the random distribution was used in our former work [20]. Set the statistical parameters, we can produce the random height distribution. Fig. 1 shows the one-dimensional Gaussian random height distribution with ξ and w taking different values. Fig. 1(a) shows the height distribution with w = 5 μm and ξ = 2 μm, Fig. 1(b) shows the distribution with w = 3 μm and ξ = 2 μm, and Fig. 1(c) is the result with w = 5 μm and ξ = 5 μm.

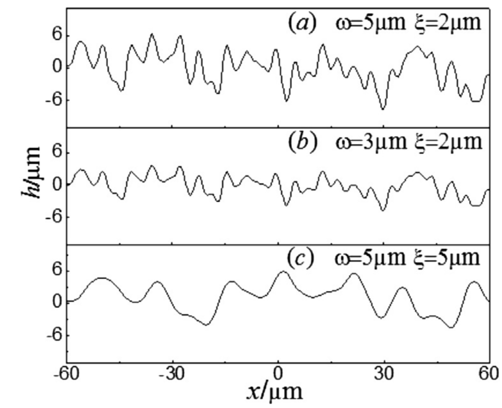

Fig. 1. Random height distribution with different statistical parameters.

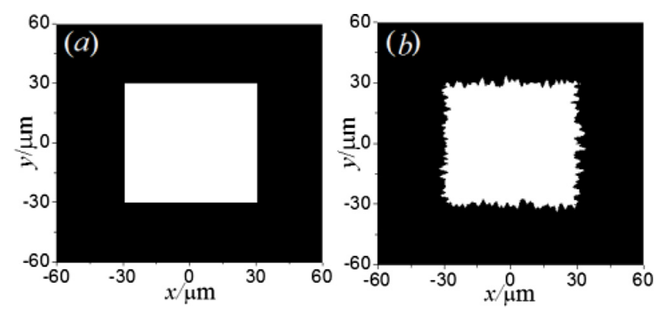

Fig. 2. Samples of perfect square apertures (a) and modulated square aperture (b).

From the results shown in Fig. 1, we can see that ξ and w reflects the rough characteristics of the edge from the directions perpendicular and parallel to the edge. Comparing the results shown in Fig. 1(a) and (b), we can see that the larger the roughness w is, the severer the longitudinal fluctuation of the edge is. From the results in Fig. 1(a) and (c), we can see that the increase of the lateral correlation length ξ results in expanding of the random distribution along the lateral direction. According to this above method, we produce a square with random edge. Fig. 2 shows the modulated square aperture with the average side length $2a$ = 60 μm, w = 2 μm and ξ = 1 μm. For convenient comparison, the perfect square aperture with the side length $2a$ = 60 μm is also shown in Fig. 2(a). It needs to point out that without loss of generality, the four random edges are different, but they have the same statistical parameters ξ and w.

For the prefect square aperture, its transmittance function can be written as $g(x, y) = g_1(x)g_2(y)$, where $g_i(\mu) = 1$ with μ limited within $[-a, a]$, otherwise $g_i(\mu) = 0$. We denote four edges including left, right, up and down sides of the modulated square aperture as $h_1(y)$, $h_2(y)$, $h_3(x)$ and $h_4(x)$. Its transmittance function can be also written as $g(x, y) = g_1(x)g_2(y)$, where $g_1(x) = 1$ with x limited within $[-a + h_1(y), a + h_2(y)]$, otherwise $g_1(x) = 0$, and $g_2(y) = 1$ with y limited within $[-a + h_3(x), a + h_4(x)]$, otherwise $g_2(y) = 0$. When a monochromatic parallel light with the wave number k (the wavelength λ) illuminates perpendicularly the square aperture placed on the plane xy, the diffraction field with the distance z from the square aperture can be written as [21],

$$u(x_1, y_1) = \frac{A}{j\lambda z}e^{jkz}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} g(x, y)\exp\left[jk\frac{(x_1 - x)^2 + (y_1 - y)^2}{2z}\right]\mathrm{d}x\mathrm{d}y \quad (5)$$

where A is a constant. According to Eq. (5), we can obtain the diffraction fields of the square apertures with smooth edge and