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Temporal and time-averaged Talbot effect of grating

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ABSTRACT

Talbot effect of grating with the ultra-short pulsed laser illumination is studied in time domain and space domain. The theoretical formulas of the temporal and time-averaged diffraction intensity of grating illuminated by the pulsed laser are deduced on basis of the partial coherence theory. The influence of the duration time of pulse and the chirp coefficient on the Talbot image of grating is analyzed in detail. As the duration time is long enough, the temporal and the time-averaged Talbot image can be obtained. Moreover, the influence of the pulse shape on Talbot effect of grating is also discussed. The diffraction distributions of grating illuminated by three kinds of pulses with Gaussian, super-Gaussian and sech shapes are provided. Under the same condition, the diffraction of grating illuminated by the pulsed laser with super-Gaussian shape closes much more to the grating structure.

1. Introduction

When a grating is illuminated by a monochromatic continuous light source, the exact self-image of the grating can be observed in the Fresnel diffraction region, and this is the so-called Talbot effect [1]. Since the Talbot effect was renewed by Lohmann [2], many extensions and applications such as interferometric measurement, phase locking of the laser array, laser array illumination, and so on, have been proposed [3–5]. Besides, with the deep study of Talbot effect in linear optics, it has extended to the regions of nonlinear optics, atomic physics and Bose Einstein condensates [6–9]. Till now, Talbot effect has been received the extensive attention of different courses. Usually, the diffraction distribution of grating can be easily obtained by use of Fresnel diffraction formula under the illumination of monochromatic continuous light. When the light source is partially coherent in time and space, Talbot effect of grating must be changed, and the partial coherence theory should be adopted to study the diffraction of grating. In our earlier work, the cross mutual spectral density was just used to study the Fresnel diffraction of grating under the illumination of any coherent light, and the effect of temporal and spatial coherence on Talbot effect of grating was discussed [10].

The ultra-short pulsed laser can reveal transient and singular phenomena among many reactions, thus the ultra-short laser becomes an important technology to research the microscopic reaction mechanism and the ultrafast process [11–13]. So far, the production of ultra-short pulsed laser and its propagation characteristics through the linear or nonlinear media have been studied by many researches [14–16]. As we know, for the ultra-short pulsed laser, the light beam contains abundant spectrum components. The wider spectral distribution will affect the quality of self-imaging of grating. Some theoretical and experimental works about the Talbot effect under femtosecond laser illumination has been performed by some researchers [10,17–21]. All these studies are based on the model of the Gaussian-shaped pulse. In fact, the pulse shape changes with the laser type. However, the self-imaging of the grating illuminated by the pulsed laser with any other shape has not been considered. In this paper, we aim to study deeply Talbot effect of grating under the illumination of the ultra-short pulsed laser with any pulse shape. Based on the partially coherent theory, the Fresnel diffraction of grating in time domain and space domain is analyzed, and the expressions of the temporal and time-averaged intensity distributions are deduced. The effect of the duration time and the chirp parameter of the pulse on the Talbot effect of grating is discussed. The simulations for the diffractions of grating illuminated by the ultra-short pulsed lasers with different parameters are performed. Section 2 provides the detailed theoretic analysis about the diffraction of grating with the ultra-short pulsed laser illumination. The time domain and spatial distributions of temporal and time-averaged diffraction intensity of grating are numerically simulated and the corresponding calculations are provided in Section 3. Section 4 provides the comparison of the diffractions of grating illuminated by Gaussian, super-Gaussian and sech pulsed lasers. Finally, the conclusions of the paper are provided.

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2. Theoretic analysis of diffraction of grating with the ultra-short pulse illumination

According to partially coherent theory [10,22,23], the cross mutual spectral density of the source at any two points on the grating plane is defined as the following

$$s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)=<\left\langle u(\mathbf{r}_{01},\omega_1)u^{*}(\mathbf{r}_{02},\omega_2)\right\rangle \tag{1}$$

where $\langle\rangle$ denotes the ensemble average and * represents the complex conjugation. We assume that the transmissivity function of grating is $T(\mathbf{r}_0)$, the cross mutual spectral density of the light wave behind immediately the grating can be expressed by $s(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)=s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)\,T(\mathbf{r}_{01})\,T^{*}(\mathbf{r}_{02})$. Then the cross mutual spectral density of the light wave at the two points on the observation plane is given by

$$s(\mathbf{r}_1,\mathbf{r}_2,\omega_1,\omega_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)T(\mathbf{r}_{01})T^{*}(\mathbf{r}_{02})\times K(\mathbf{r}_1;\mathbf{r}_{01},\omega_1)K^{*}(\mathbf{r}_2;\mathbf{r}_{02},\omega_2)d\mathbf{r}_{01}d\mathbf{r}_{02} \tag{2}$$

where $K(\mathbf{r};\mathbf{r}_0,\omega)$ is the point spread function of the Fresnel diffraction system. Under the paraxial approximation, it can be simplified into the following form,

$$K(\mathbf{r};\mathbf{r}_0,\omega)=\frac{\omega}{i2\pi cz}\exp\left(\frac{i\omega z}{c}+\frac{i\omega}{2zc}|\mathbf{r}-\mathbf{r}_0|^2\right) \tag{3}$$

where c is the velocity of the light in vacuum, and z is the distance between the grating and the observation plane. By using the relation between the cross mutual spectral density and the mutual coherent function, we can express the space–time correlation functions $\Gamma(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)$ at two points on the observation plane as

$$\Gamma(\mathbf{r}_1,\mathbf{r}_2,t_1,t_2)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s(\mathbf{r}_1,\mathbf{r}_2,\omega_1,\omega_2)\exp[i(\omega_1t_1-\omega_2t_2)]d\omega_1d\omega_2 \tag{4}$$

Then, the temporal diffractive intensity distribution of the grating at one point in the Fresnel diffraction region $I(\mathbf{r},t)=\Gamma(\mathbf{r}_1,\mathbf{r}_2,t,t)$can be obtained

$$I(\mathbf{r},t)=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)\frac{\omega_1\omega_2}{4\pi^2c^2z^2}\times T(\mathbf{r}_{01})T^{*}(\mathbf{r}_{02})\exp\left\{i\left[\left(\frac{\omega_1|\mathbf{r}-\mathbf{r}_{01}|^2-\omega_2|\mathbf{r}-\mathbf{r}_{02}|^2}{2zc}\right)+(\omega_1-\omega_2)\left(t-\frac{z}{c}\right)\right]\right\}d\mathbf{r}_{01}d\mathbf{r}_{02}d\omega_1d\omega_2 \tag{5}$$

And the time-averaged diffractive intensity distribution can be got by the integration of the temporal diffractive intensity distribution over time,

$$I(\mathbf{r})=\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega,\omega)\frac{\omega^2}{4\pi^2c^2z^2}T(\mathbf{r}_{01})T^{*}(\mathbf{r}_{02})\exp\left[\frac{i\omega}{2cz}(|\mathbf{r}_1-\mathbf{r}_{01}|^2-|\mathbf{r}_1-\mathbf{r}_{02}|^2)\right]d\mathbf{r}_{01}d\mathbf{r}_{02}d\omega \tag{6}$$

For different light source, $s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)$ takes the different form. When the light source is coherent in space and partially coherent in time, such as the uniform ultra-short pulsed laser beam, the mutual spectral density of the light source can be written as

$$s^{-}(\mathbf{r}_{01},\mathbf{r}_{02},\omega_1,\omega_2)=s^{-}(\omega_1,\omega_2)=\left\langle u(\omega_1)u^{*}(\omega_2)\right\rangle \tag{7}$$

where $u(\omega_i)$ is the spectral distribution of ultra-short pulsed laser. While the incident pulse is Gaussian shaped, the temporal distribution of the pulse $u(t)$ has the following form,

$$u(t)=\exp(-i\omega_0t)\exp\left[-\frac{(1+iC)t^2}{2\tau^2}\right] \tag{8}$$

where ω_0 is the central frequency, called the carrier frequency of the laser, τ is the effective duration time of pulse, and C is the pulse chirp parameter. When $C=0$, the pulse is not chirped. As $C>0$, it is positively chirped, and contrarily, as $C<0$, it is negatively chirped. The spectrum of the chirped pulse can be obtained through the Fourier transform,

$$u(\omega)=\frac{\tau}{\sqrt{2\pi(1+iC)}}\exp\left[-\frac{(\omega-\omega_0)^2\tau^2}{2(1+iC)}\right] \tag{9}$$

From this above formula, we can see that the effective duration time τ is shorter, the spectral distribution is wider. Insert Eq. (9) into Eq. (7), the spectral density of the pulsed laser with Gaussian shape can be obtained,

$$\begin{aligned}s^{-}(\omega_1,\omega_2)&=\frac{\tau^2}{2\pi\sqrt{1+C^2}}\exp\left\{\frac{-[(\omega_1-\omega_0)^2+(\omega_2-\omega_0)^2]\tau^2}{2(1+C^2)}\right\}\\&\times\exp\left\{\frac{iC[(\omega_1-\omega_0)^2-(\omega_2-\omega_0)^2]\tau^2}{2(1+C^2)}\right\}\end{aligned} \tag{10}$$

Here, we choose one-dimensional transmission rectangular grating as an example to study Talbot effect of grating. Its transmissivity function can be written as,

$$T(x_0)=\sum_{n=0}^{N}\mathrm{rect}\left(\frac{x_0-nd}{d/M}\right) \tag{11}$$

where d is the grating period, and $M=a/d$ (a is the opening size) is the duty cycle of grating. Since this function is integrated absolutely in the interval of $[-d/2,d/2]$, this periodic function can be spread into Fourier series,

$$T(x_0)=\sum_{n=-\infty}^{\infty}A_n\exp\left(\frac{i2\pi nx_0}{d}\right) \tag{12}$$

where A_n is Fourier coefficient. Insert Eq. (12) into Eq. (5) and Eq. (6) and perform the integral in space domain, the expressions of one-dimensional temporal and time-averaged diffraction intensity distributions of the grating with the pulsed laser illumination can be written as

$$\begin{aligned}I(x,z,t)=&\left|\int_{-\infty}^{\infty}\frac{\sqrt{\omega}\tau}{2\pi\sqrt{(1+C^2)}cz}\exp\left[-\frac{(\omega-\omega_0)^2\tau^2}{2(1+C^2)}\right]\right.\\&\times\exp\left[\frac{iC(\omega-\omega_0)^2\tau^2}{2(1+C^2)}\right]\exp\left[i\omega\left(t-\frac{z}{c}\right)\right]\\&\left.\times\sum_m A_m\exp\left(\frac{i2\pi xm}{d}\right)\exp\left(-i\frac{2\pi^2zcm^2}{d^2\omega}\right)d\omega\right|^2\end{aligned} \tag{13}$$

and

$$\begin{aligned}I(x,z)=&\int_{-\infty}^{\infty}\frac{\tau^2}{2\pi\sqrt{1+C^2}}\exp\left[-\frac{(\omega-\omega_0)^2\tau^2}{1+C^2}\right]\frac{\omega}{2\pi cz}\\&\times\left|\sum_m A_m\exp\left(\frac{i2\pi xm}{d}\right)\exp\left(-i\frac{2\pi^2zcm^2}{d^2\omega}\right)\right|^2d\omega\end{aligned} \tag{14}$$

Thus, we can obtain the temporal and time-averaged intensity of grating at any observation point with the Gaussian pulsed laser illumination. According to Eqs. (13) and (14), we can also see the diffraction intensity at one point is the contribution of all spectral components, and it depends on the duration time τ and the chirp parameter C of pulse. In the following section, we give the numerical calculations of the temporal and time-averaged diffraction intensities. In practical calculations, we set the diffraction orders 80 and the frequency width $\Delta w/w_0=1.27$ ($\Delta\lambda/\lambda_0=1.25$) with the carrier frequency $\omega_0=2.355\times1015$ Hz.