

# Mutual focusing of two laser beams propagating in homogeneous plasma

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## ABSTRACT

In the present paper, self-focusing of two laser beams, having equal frequency (wave number), propagating in cold, homogeneous and underdense plasma has been studied. The polarization vectors of the two beams are considered to be arbitrarily oriented, and lying in a plane perpendicular to the propagation direction. Using source dependent expansion technique, the evolution equations for spot size and phase shift of the laser beams having radial Gaussian profile, is derived. Numerical solutions for simultaneous evolution of the laser spot of the two beams have been obtained.

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## 1. Introduction

Propagation of intense laser beams in plasma has applications in nuclear fusion [1,2], laser wakefield acceleration [3,4] and X-ray lasers [5,6]. At high laser intensity, nonlinear interaction between laser and plasma becomes important. An important nonlinear process arising from laser–plasma interaction is self-focusing of the laser beam. Much theoretical [7,8] and experimental [9,10] progress regarding self-focusing of the laser light in plasma has been reported. Physically, relativistic self-focusing arises from the relativistic quiver velocity  $v_q (= ca/\gamma)$  in the radiation field, where  $\mathbf{a} (= e\mathbf{E}/mc\omega)$  is the normalized radiation field and  $\gamma$  is the relativistic factor for an electron in a radiation field. The focusing mechanism for a single beam is that a radiation profile peaked on-axis leads to an index of refraction profile  $n \sim 1 - (\omega_p/\omega)^2/2\gamma$ , which has a minimum on axis [11]. Therefore the radiation beam focuses close to the axis.

The presence of multiple beams in plasma can give rise to a new set of interesting phenomena. One of the potential applications of two colliding beams in plasma is the excitation of large amplitude electron plasma waves (EPW), which in turn accelerate electrons to ultra relativistic speeds [12]. Laser beams that are co-propagating at a small distance from each other sometimes spiral around each other [13]. At relativistic intensities, laser beams can give rise to fast plasma waves via higher order nonlinearities [13–15] or via beat wave excitation at frequencies different from the electron

plasma frequency [16]. It has recently been reported that stability and reproducibility of electron acceleration can be improved by counter-propagating laser pulses [17]. An increase in number of hot electrons and hot electron temperature has been observed due to two crossed laser beams propagating in plasma, by Zhang et al. [18]. The reduction of the amplitude of ion acoustic waves associated with stimulated Brillouin scattering and amplification of electron plasma waves (EPW) associated with stimulated Raman scattering for multiple beams are reported by Labaune et al. [19]. These two instabilities have important implications in ICF due to possible large conversion of laser energy into scattered light affecting ICF drive efficiency and symmetry.

The combined representation of the normalized electric field of two beams can be written as  $\mathbf{a} = \mathbf{a}_1 + \mathbf{a}_2$  and the resultant intensity is given by  $|\mathbf{a}|^2 = |\mathbf{a}_1|^2 + |\mathbf{a}_2|^2 + \mathbf{a}_1 \cdot \mathbf{a}_2^* + \mathbf{a}_1^* \cdot \mathbf{a}_2$ , where  $\mathbf{a}_1 \cdot \mathbf{a}_2^*$  and  $\mathbf{a}_1^* \cdot \mathbf{a}_2$  are the cross terms. Some of the earlier studies on self-focusing of two beams (of different frequencies) have considered the beat wave process for which the cross terms have been neglected. These terms do not appear due to averaging over beat wave phase [20] or the configuration is such that electric vectors of two beams are mutually perpendicular [21]. Thus the interaction dynamics is restricted to incoherent cases. Recently, Dong et al. [22] have reported a numerical analysis of mutual interaction of two parallel polarized beams of equal frequency propagating in plasma.

In the present paper, self-focusing of two laser beams having equal frequency, in uniform plasma, is studied theoretically. The polarization vectors of the two beams are considered to be arbitrarily oriented, and the role of cross terms has been included. The laser pulse length has been taken large ( $L \gg \lambda_p$ ) so that finite pulse length and group velocity dispersion effects can be neglected and a

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paraxial analysis is sufficient to describe the propagation dynamics of the two laser beams.

The paper is organized as follows: In Section 2, the wave equation describing the evolution of two laser beams in the weakly relativistic regime has been set up. Considering appropriate trial functions and using source dependent expansion technique, the evolution equations for spot size, phase shift and curvature of the two beams, are obtained in Section 3. Section 4 depicts the evolution of the spot sizes with propagation distance for different laser and plasma parameters, using numerical techniques. Summary and conclusions are discussed in Section 5.

## 2. Wave dynamics

Consider two linearly polarized laser beams propagating in uniform homogenous plasma of density  $n_0$ . The electric vectors of radiation fields propagating along the  $z$ -direction are represented by

$$\mathbf{E}_1 = \hat{e}_1 E_1(r, z, t) \exp i(kz - \omega t) + c \cdot c \quad (1)$$

and

$$\mathbf{E}_2 = \hat{e}_2 E_2(r, z, t) \exp i(kz - \omega t) + c \cdot c \quad (2)$$

where  $\hat{e}_1$  and  $\hat{e}_2$  are arbitrary polarization directions for the first and second beams respectively, lying in a plane perpendicular to the propagation direction  $z$ . The frequency ( $\omega$ ) and wave number ( $k$ ) of the two beams are the same, while the amplitudes ( $E_1$  and  $E_2$ ) are different.

The wave equations governing the propagation of the laser beams are given by

$$\nabla^2 \mathbf{E}_i - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_i}{\partial t^2} = \frac{k_{po}^2 \mathbf{E}_i}{\gamma} \quad (3)$$

where  $i = 1, 2$ ,  $\gamma = \left(1 - (v^2/c^2)\right)^{-1/2}$  is the relativistic factor and  $\mathbf{v} = (\mathbf{v}_1 + \mathbf{v}_2)$  is the combined quiver velocity of the plasma electrons in the presence of the two radiation fields.

The quiver velocity of the plasma electrons can be obtained from the Lorentz force equation

$$\frac{\partial \mathbf{v}_i}{\partial t} = -\frac{e\mathbf{E}}{m} \quad (4)$$

Thus

$$\mathbf{v} = \hat{e}_1 c a_1 \exp i(kz - \omega t) + \hat{e}_2 c a_2 \exp i(kz - \omega t) + c \cdot c \quad (5)$$

where  $a_i (= eE_i/mc\omega)$  is the normalized electric field amplitude. Assuming  $a_i < 1$ , the combined relativistic factor due to two beams is given by

$$\gamma = 1 + \frac{a_1^2}{4} + \frac{a_2^2}{4} + \frac{e_1 \cdot e_2}{4} (a_1^* a_2 + a_1 a_2^*) \quad (6)$$

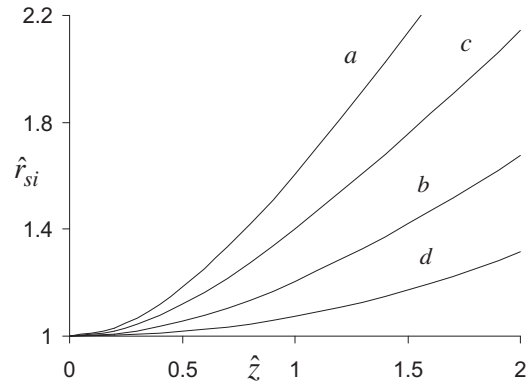
Substituting the value of  $\gamma$ , the wave equation governing the propagation of the two beams is given by

$$\nabla^2 \mathbf{E}_i - \frac{1}{c^2} \frac{\partial^2 \mathbf{E}_i}{\partial t^2} = k_{po}^2 \left[ 1 - \frac{a_1^2}{4} - \frac{a_2^2}{4} - \frac{e_1 \cdot e_2}{4} (a_1^* a_2 + a_1 a_2^*) \right] \mathbf{E}_i \quad (7)$$

Substituting Eq. (1) into Eq. (7) and neglecting higher order diffraction effects, the paraxial wave equation is given by

$$\left( \nabla_{\perp}^2 + 2ik \frac{\partial}{\partial z} \right) a_i = -k_{po}^2 a_i \left[ \frac{a_1^2}{4} + \frac{a_2^2}{4} + \hat{e}_1 \cdot \hat{e}_2 (a_1^* a_2 + a_1 a_2^*) \right] \quad (8)$$

The right side of Eq. (8) shows that the source driving each of the two beam amplitudes is due to a nonlinear combination of both beam amplitudes.



**Fig. 1.** Variation of normalized spot size with normalized propagation distance of two identical (same frequency) laser beams with equal intensity ( $a_{10} = a_{20} = 0.3$ ) and equal initial spot size ( $r_{10} = r_{20} = 20 \mu\text{m}$ ) for parallel (curve (a)), perpendicular (curve (b)), antiparallel (curve (c)) polarization and a single beam (curve (d)) having the same intensity and spot size for  $\lambda_p = 15 \mu\text{m}$ .

## 3. Equations for spot sizes

In order to obtain the spot size evolution of the two beams, the source dependent expansion (SDE) technique [23] is used. The field amplitude of the each laser beam is expanded as a series of Laguerre–Gaussian source dependent modes and may be written as

$$a_i(r, z) = \sum_m a_{im}(z) L_m(\chi_i) \exp \left[ \frac{i\theta_i - (1 - i\alpha_i)\chi_i}{2} \right], \quad (9)$$

where  $m = 0, 1, 2, 3, \dots$ ,  $a_{im}(z)$  is the complex amplitude,  $\chi_i = 2r^2/r_{si}^2$ ,  $r_{si}(z)$  is the spot size,  $\alpha_i(z)$  is related to the curvature associated with the wavefront,  $L_m(\chi_i)$  is a Laguerre polynomial of order  $m$  and  $\theta_i$  is the phase shift. The amplitude of the lowest order mode ( $m = 0$ ) for each of the two beams is assumed to be

$$a_i = a_{si} \exp \left[ \frac{i\theta_i - (1 - i\alpha_i)r^2}{r_{si}^2} \right] \quad (10)$$

The evolution of real parameters  $a_{si}(z)$ ,  $r_{si}(z)$ ,  $\alpha_i(z)$  and  $\theta_i(z)$  for the lowest (Gaussian) mode is given by

$$\frac{\partial}{\partial z} (a_{si} r_{si}) = 0 \quad (11)$$

$$\frac{\partial^2 r_{si}}{\partial z^2} = \frac{4(1 + k r_{si}^2 H_i)}{k^2 r_{si}^3} \quad (12)$$

$$\alpha_i = \frac{k r_{si}}{2} \frac{\partial r_{si}}{\partial z} \quad (13)$$

and

$$\frac{\partial \theta_i}{\partial z} = -\frac{2}{k r_{si}^2} - H_i - G_i \quad (14)$$

where

$$G_i = \frac{k}{2} \int_0^\infty (1 - \eta^2) \exp(-\chi_i) d\chi_i \quad (15)$$

$$H_i = \frac{k}{2} \int_0^\infty (1 - \eta^2) (1 - \chi_i) \exp(-\chi_i) d\chi_i \quad (16)$$

$$\text{and } 1 - \eta^2 = -\frac{k_{po}^2}{k^2} \left[ \frac{a_1^2}{4} + \frac{a_2^2}{4} + \hat{e}_1 \cdot \hat{e}_2 (a_1^* a_2 + a_1 a_2^*) \right]$$

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