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Adaptive modified generalized function projection synchronization between integer-order and fractional-order chaotic systems

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ABSTRACT

This paper investigates modified general functional projective synchronization (MGFPS) between a class of integer-order and fractional-order chaotic systems. For the parameters in the chaotic system known and unknown, two adaptive control schemes are presented respectively to make the drive and response systems synchronized. Numerical simulations are given to illustrate the effectiveness of the theoretical results.

1. Introduction

Chaotic synchronization has received considerable attention in recent years due to its potential applications in a variety of fields, such as secure communication, chemical reaction, information processing, biological systems and so on [1–8]. So far many synchronization schemes are presented to synchronize chaotic systems, including complete synchronization, general synchronization, phase synchronization, lag synchronization, intermittent lag synchronization, projection synchronization, function projection synchronization, and so forth. Amongst these schemes function projection synchronization (FPS) is widely used in the field of secure communication as it ensures drive and response systems synchronized up to a desired scaling function, which may greatly strengthen the security of communication [9,10]. Based on this, recently, generalized function projection synchronization(GFPS) has also been proposed [11]. In GFPS, drive and response systems synchronized up to a desired scaling function matrix, which is more general than FPS.

Fractional-order chaotic system and its synchronization attract increasing attention due to its potential applications in secure communication. In recent decades, many researchers have investigated synchronization of fractional-order chaotic systems, for instance, the fractional-order Lorenz, Chen, Lü and Rössler systems etc, see [12,13] and reference therein. As to the synchronization between integer-order and fractional-order chaotic systems, there are relatively few works concerning this aspect. In this paper, we propose modified generalized function projection synchronization (MGFPS) between a class of integer-order and fractional-order chaotic systems.

The rest of this paper is structured as follows. In Section 2, a class of unified chaotic system is presented and the MGFPS between integer-order and fractional-order chaotic systems is discussed. For the systems parameters known and unknown, two simple adaptive control schemes are provided to make the drive system and the response system synchronized. In Section 3, some numerical simulations are carried out to show the effectiveness of the theoretical results. Finally, in Section 4, some concluding remarks are given.

2. System description and adaptive control schemes for the synchroniztion

We consider a class of unified chaotic system described by

$$\begin{cases} \dot{x} = yf(x,y,z) + z\Phi(x,y,z) - \alpha x, \\ \dot{y} = g(x,y,z) - \beta y, \\ \dot{z} = yh(x,y,z) - x\Phi(x,y,z) - \gamma z, \end{cases} \tag{1}$$

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where x, y and z are state variables and α, β, γ are parameters. Each of four functions ($f(\cdot)$, $g(\cdot)$, $h(\cdot)$ and $\Phi(\cdot)$) is smooth in $R^3 \to R$ space. Note that many well-known chaotic systems, such as Lorenz system, Chen system, and Rössler system, can be expressed by system (1).

We choose the corresponding fractional-order system as the drive system, which is presented by

$$\begin{cases} D^{q_1}x_1 = x_2 f(x_1, x_2, x_3) + x_3\Phi(x_1, x_2, x_3) - \alpha x_1, \\ D^{q_2}x_2 = g(x_1, x_2, x_3) - \beta x_2, \\ D^{q_3}x_3 = x_2 h(x_1, x_2, x_3) - x_1\Phi(x_1, x_2, x_3) - \gamma x_3, \end{cases} \tag{2}$$

where $0 < q_i < 1 (i = 1, 2, 3)$ is the derivative order of state variable x_1, x_2 and x_3, respectively. The Caputo definition of fractional derivative is adopted, i.e., $D^{\alpha}z(x) = 1/\Gamma(\alpha)\int_0^x (x-t)^{\alpha-1}z(t)dt$, $0 < \alpha < 1$. Here $\Gamma(\cdot)$ is the gamma function.

The response system is presented in the following form

$$\begin{cases} \dot{y}_1 = y_2 f(y_1, y_2, y_3) + y_3\Phi(y_1, y_2, y_3) - \alpha y_1 + u_1, \\ \dot{y}_2 = g(y_1, y_2, y_3) - \beta y_2 + u_2, \\ \dot{y}_3 = y_2 h(y_1, y_2, y_3) - y_1\Phi(y_1, y_2, y_3) - \gamma y_3 + u_3. \end{cases} \tag{3}$$

The synchronization errors are given by

$$e_i(t) = y_i(t) - M_i(t)x_i(t), \quad i = 1, 2, 3. \tag{4}$$

Definition 1. System (2) and system (3) are said to achieve MGFPS if $\lim_{t\to\infty} \|y(t) - \Lambda x(t)\| = 0$, where $\Lambda = diag(M_1(t), M_2(t), M_3(t))$, $x(t) = (x_1(t), x_2(t), x_3(t))^T$, $y(t) = (y_1(t), y_2(t), y_3(t))^T$.

Remark 1. If $M_1(t) = M_2(t) = M_3(t)$, then MGFPS is simplified to generalized function projection synchronization (GFPS).

Substituting system (2) and (3) into Eq. (4) gives the error dynamics

$$\begin{cases} \dot{e}_1 = (y_2 f + y_3\Phi - \alpha y_1 + u_1) - \dot{M}_1x_1 - M_1D^{1-q_1}(x_2 f + x_3\Phi - \alpha x_1) \\ \dot{e}_2 = (g - \beta y_2 + u_2) - \dot{M}_2x_2 - M_2D^{1-q_2}(g - \beta x_2), \\ \dot{e}_3 = (y_2 h - y_1\Phi - \gamma y_3 + u_3) - \dot{M}_3x_3 - M_3D^{1-q_3}(x_2 h - x_1\Phi - \gamma x_3). \end{cases} \tag{5}$$

We choose the controller described as follows

$$\begin{cases} u_1 = -y_2 f - x_3M_3\Phi + \alpha M_1x_1 + \dot{M}_1x_1 + M_1D^{1-q_1}(x_2 f + x_3\Phi - \alpha x_1) - k_1(t)e_1 \\ u_2 = -g + \beta M_2x_2 + \dot{M}_2x_2 + M_2D^{1-q_2}(g - \beta x_2) - k_2(t)e_2, \\ u_3 = -y_2 h + x_1M_1\Phi + \gamma M_3x_3 + \dot{M}_3x_3 + M_3D^{1-q_3}(x_2 h - x_1\Phi - \gamma x_3) - k_3(t)e_3. \end{cases} \tag{6}$$

The adaptive law is

$$\dot{k}_i(t) = \lambda_i e_i^2, \quad i = 1, 2, 3. \tag{7}$$

where $\lambda_i (i = 1, 2, 3)$ are positive parameters controlling the convergence speed of synchronization errors.

Denote by $\tilde{k}_i(t) = k_i(t) - k_i^*$, where k_i^* is a positive constant that should be determined. Then we have the following result.

Theorem 1. *The MGFPS between system (2) and system (3) can be achieved if the controller (6) and the adaptive law (7) are applied.*

Proof. Construct the Lyapunov candidate as follows:

$$V(t) = \frac{1}{2}\sum_{i=1}^{3} e_i^2 + \frac{1}{2}\sum_{i=1}^{3}\frac{\tilde{k}_i^2(t)}{\lambda_i}. \tag{8}$$

Taking the time derivative of $V(t)$ along the trajectories of the synchronization error dynamical system (5) and applying the adaptive law (7), one has

$$\begin{aligned} \dot{V}(t) &= e_1(-\alpha e_1 + \Phi e_3 - k_1e_1) + e_2(-\beta e_2 - k_2e_2) + e_3(-\gamma e_3 - \Phi e_1 - k_3e_3) + (k_1 - k_1^*)e_1^2 + (k_2 - k_2^*)e_2^2 + (k_3 - k_3^*)e_3^2 \\ &= (-\alpha - k_1^*)e_1^2 + (-\beta - k_2^*)e_2^2 + (-\gamma - k_3^*)e_3^2. \end{aligned} \tag{9}$$

It is obvious that there exist sufficiently large positive constants $k_i^* (i = 1, 2, 3)$ such that $-\alpha - k_1^* < 0$, $-\beta - k_2^* < 0$, $-\gamma - k_3^* < 0$, and then one has $\dot{V}(t) < 0$, which implies $\lim_{t\to\infty}\|y_i(t) - M_i(t)x_i(t)\| = 0, \quad i = 1, 2, 3$. Thus the MGFPS between system (2) and system (3) can be achieved. □

Note that the above result is discussed under the condition that system parameters are known. While in real applications, it is often subject to unknown system parameters. With regard to this case, we choose the response system presented as follows:

$$\begin{cases} \dot{y}_1 = y_2 f(y_1, y_2, y_3) + y_3\Phi(y_1, y_2, y_3) - \tilde{\alpha} y_1 + u_1', \\ \dot{y}_2 = g(y_1, y_2, y_3) - \tilde{\beta} y_2 + u_2', \\ \dot{y}_3 = y_2 h(y_1, y_2, y_3) - y_1\Phi(y_1, y_2, y_3) - \tilde{\gamma} y_3 + u_3', \end{cases} \tag{10}$$