



# Vision pose estimation from planar dual circles in a single image



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## ABSTRACT

A novel vision based pose estimation method for a single image with planar dual circles is addressed. We present a very simple formula to solve camera pose with a single circle, and then develop a fusion method to integrate solved poses of dual circles by using space geometry constraints. After that, a new definition of the difference quantity between two ellipses is proposed to evaluate reprojection errors of dual circles and determine the optimal and unique pose solution. Experiments with synthetic data and real images are carried out to validate the proposed method, and results show that the method has a high accuracy and good robustness.

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## 1. Introduction

Pose estimation of camera from a single image is a basic and important problem in computer vision literature for the last two decades [1,2]. The well-known perspective-n-point (PnP) problem, which was first proposed by Fishler and Bolles, is to find the pose of an object from the image of  $n$  points at known location on it [3,4]. A large number of absolute pose algorithms for PnP problem have been studied [5,6]. Collins presented a new analytic solution to the problem which is far faster than current methods based on solving PnP problem [7]. Zhou proposed the SoftSI algorithm to obtain pose and correspondences simultaneously, which is based on the combination of the proposed PnP algorithm (the SI algorithm) and two SVD-based shape description theorems [8]. Line correspondences were also commonly used to estimate the camera pose, since lines are very similar with points in perspective transformation [9].

Another kind of patterns in man-made objects and scenes are circular features, which have been widely applied to camera calibration, autonomous navigation and industrial detection [10,11]. Under perspective geometry, a projected circle appears as an ellipse in the imaging plane, and the 3D position of the circle can be extracted from single image using the inverse projection model of the calibrated camera [12,13]. Zhao used projected circle centers to calibrate camera parameters [14], and Lu presented a mono-circular-vision-based method for localization of underwater circular features [15].

However, two possible pose solutions can be recovered under normal circumstances when employing a single circle. External information should be added to determine the unique pose solution [16,17]. Such as, three non-concentric circles [18], two parallel

circles [19], or two arbitrary coplanar circles [20] were used as calibration patterns. Ma presented a circle pose estimation method based on binocular stereo vision [21]. Rahmann proposed a minimal and linear solver from two views using two arbitrary circles [22]. Gurdjos stated the recovering problem of 2D Euclidean structure from  $N$  parallel circles in terms of a system of linear equations to solve and provided a closed-form solution [23]. Another problem of the circular vision is the difficulty to evaluate pose estimation results, since previous methods are based on analytical geometry.

In this paper, we propose a method to determine the exact camera pose using dual circle with different centers in a same plane. Unlike the existed method using the concept of the absolute conic to solve the relative pose based on only a single circle, we deduct a very simple formula to solve the single circle based pose determination problem. And then we present a fusion method to integrate poses of dual circles, and define a new difference quantity of two ellipses in order to evaluate reprojection errors of circles and determine the unique and optimal pose estimation solution.

This paper is organized as follows. Section 2 briefly introduces some notations and basic equations in computer vision. Section 3 derives a very simple formula to solve the pose determination problem based on a single circle. After the pose for single circle is known, dual circles based vision pose estimation method is induced in Section 4. The results of experiments with synthetic data and real images are shown in Sections 5 and 6. Finally, the conclude is presented in Section 7.

## 2. Perspective geometry and circular projection

### 2.1. Camera projection model

Let  $\tilde{\mathbf{x}} = (x, y, z, 1)^T$  be the 3D homogeneous coordinates of a world 3D point  $\mathbf{x} \in \mathbb{R}^3$ , and  $\tilde{\mathbf{m}} = (u, v, 1)^T$  be the homogeneous

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coordinates of its projection in the image plane. Under perspective geometry, the projection relationship can be described as:

$$z_c \tilde{\mathbf{m}} = \mathbf{K} \mathbf{X}_c = \mathbf{K} [\mathbf{R} \ \mathbf{t}] \tilde{\mathbf{x}} \quad \text{with} \quad \mathbf{K} = \begin{bmatrix} f_u & 0 & u_0 \\ 0 & f_v & v_0 \\ 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where  $\mathbf{X}_c = (x_c, y_c, z_c)^T$  is the 3D coordinates of  $\mathbf{x}$  in camera frame.  $\mathbf{K}$  is the camera intrinsic matrix, consisted of the principle point  $(u_0, v_0)$  and the focal length  $(f_u, f_v)$ .  $[\mathbf{R} \ \mathbf{t}]$  is the camera extrinsic matrix, where  $\mathbf{R} = [\mathbf{r}_1 \ \mathbf{r}_2 \ \mathbf{r}_3]$  means the rotation matrix, and  $\mathbf{t}$  stands for the translation vector from the world frame to the camera frame.

### 2.2. Projection of a circle

Without loss of generality, we may assume the world space is restricted to its  $x$ - $y$  plane, which means  $z=0$ . Then a point  $\mathbf{x}=(x, y, 0)^T$  on a circle  $\mathbf{C}$  satisfies the following equation:

$$\tilde{\mathbf{x}}_p^T \mathbf{P} \tilde{\mathbf{x}}_p = 0 \quad \text{with} \quad \mathbf{P} = \begin{bmatrix} 1 & 0 & -x_0 \\ 0 & 1 & -y_0 \\ -x_0 & -y_0 & x_0^2 + y_0^2 - r^2 \end{bmatrix} \quad (2)$$

where  $\tilde{\mathbf{x}}_p = (x, y, 1)^T$ ,  $(x_0, y_0)$  is the center of  $\mathbf{C}$ , and  $r$  is the radius of  $\mathbf{C}$ .

Substituting  $z=0$  into Eq. (1), then we can get

$$\tilde{\mathbf{x}}_p = z_c \mathbf{H}^{-1} \tilde{\mathbf{m}} \quad \text{with} \quad \mathbf{H} = \mathbf{K} [\mathbf{r}_1 \ \mathbf{r}_2 \ \mathbf{t}] \quad (3)$$

With Eqs. (2) and (3), the projected ellipse  $\mathbf{E}$  of the circle  $\mathbf{C}$  can be expressed as

$$\tilde{\mathbf{m}}^T \mathbf{Q} \tilde{\mathbf{m}} = 0 \quad \text{with} \quad \mathbf{Q} = \mu \mathbf{H}^{-T} \mathbf{P} \mathbf{H}^{-1} \quad (4)$$

where  $\mu$  is a non-zero factor.

### 3. Vision pose solver with single circle

If the camera is assumed calibrated, we may set  $\mathbf{K} = \mathbf{I}_3 = \text{diag}(1, 1, 1)$ , which is equivalent to normalizing the image coordinates by applying transformation  $\mathbf{K}^{-1}$ . Then

$$\mathbf{H} = [\mathbf{r}_1 \ \mathbf{r}_2 \ \mathbf{t}] = \mathbf{R} [\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{s}] \mathbf{R}^T \mathbf{t} \quad \text{and} \quad \mathbf{H}^{-1} = [\mathbf{e}_1 \ \mathbf{e}_2 \ \mathbf{s}] \mathbf{R}^T \quad (5)$$

where  $\mathbf{e}_1 = (1, 0, 0)^T$ ,  $\mathbf{e}_2 = (0, 1, 0)^T$ , and  $\mathbf{s} = (s_1, s_2, s_3)^T$  satisfies

$$\mathbf{R}^T \mathbf{t} = \begin{pmatrix} -s_1 \\ -s_2 \\ \frac{1}{s_3} \end{pmatrix}^T \quad \text{or} \quad \mathbf{t} = -\frac{1}{s_3} \mathbf{R} \cdot (s_1, s_2, -1)^T \quad (6)$$

Since  $-\mathbf{R}^T \mathbf{t}$  represents the optic center position coordinate in the world frame, we have  $s_3 < 0$  when the  $z$ -axis of the world frame is set pointed to the camera.

If we set the center of the circle as the world frame's origin point, then  $\mathbf{P}$  in Eq. (2) can be written as  $\mathbf{P} = \text{diag}(1, 1, -r^2)$ , and  $\mathbf{Q}$  is obtained from Eqs. (4) and (5) as

$$\mathbf{Q} = \mu \mathbf{R} \mathbf{M} \mathbf{R}^T \quad \text{with} \quad \mathbf{M} = \begin{bmatrix} 1 & 0 & s_1 \\ 0 & 1 & s_2 \\ s_1 & s_2 & s_1^2 + s_2^2 - s_3^2 r^2 \end{bmatrix} \quad (7)$$

Diagonalizing  $\mathbf{M}$  as  $\mathbf{M} = \mathbf{U} \cdot \text{diag}(\eta_1, \eta_2, \eta_3) \cdot \mathbf{U}^T$ , we have

$$\eta_1 = 1, \quad \eta_2 + \eta_3 = 1 + s_1^2 + s_2^2 - s_3^2 r^2, \quad \eta_2 \eta_3 = -s_3^2 r^2 \quad (8)$$

$$\mathbf{U}^T = \begin{bmatrix} \frac{-s_2}{\sqrt{s_1^2 + s_2^2}} & \frac{s_1}{\sqrt{s_1^2 + s_2^2}} & 0 \\ \frac{\sqrt{s_1^2 + s_2^2} + (\eta_2 - 1)}{s_1} & \frac{\sqrt{s_1^2 + s_2^2} + (\eta_2 - 1)}{s_2} & \frac{\eta_2 - 1}{\sqrt{s_1^2 + s_2^2 + (\eta_2 - 1)^2}} \\ \frac{\sqrt{s_1^2 + s_2^2} + (\eta_3 - 1)}{\sqrt{s_1^2 + s_2^2 + (\eta_3 - 1)^2}} & \frac{\sqrt{s_1^2 + s_2^2} + (\eta_3 - 1)}{\sqrt{s_1^2 + s_2^2 + (\eta_3 - 1)^2}} & \frac{\eta_3 - 1}{\sqrt{s_1^2 + s_2^2 + (\eta_3 - 1)^2}} \end{bmatrix} \quad (9)$$

and  $\det(\mathbf{U}) = 1$ . It is obvious that one of  $(\eta_2, \eta_3)$  is positive and another is negative from Eq. (8). Let us assume  $\eta_2 > 0 > \eta_3$ , and then the order of  $(\eta_1, \eta_2, \eta_3)$  is determined by using  $(1 - \eta_2)(1 - \eta_3) = -(s_1^2 + s_2^2) \leq 0$  as

$$\eta_2 \geq \eta_1 = 1 > 0 > \eta_3 \quad (10)$$

We can also diagonalize  $\mathbf{Q}$  as  $\mathbf{Q} = \mathbf{V} \cdot \text{diag}(\lambda_1, \lambda_2, \lambda_3) \cdot \mathbf{V}^T$ , and assure  $\det(\mathbf{V}) = 1$ , and the order of  $(\lambda_1, \lambda_2, \lambda_3)$  as  $\lambda_2 \geq \lambda_1 \geq \lambda_3$  if  $\lambda_1 \lambda_2 \lambda_3 > 0$ , or  $\lambda_2 \leq \lambda_1 \leq \lambda_3$  if  $\lambda_1 \lambda_2 \lambda_3 < 0$ . Further, we can establish the relationship between  $\mathbf{M}$  and  $\mathbf{Q}$  through Eq. (7) as

$$(\lambda_1, \lambda_2, \lambda_3)^T = \mu (\eta_1, \eta_2, \eta_3)^T \quad \text{and} \quad \eta_2 = \frac{\lambda_2}{\lambda_1}, \eta_3 = \frac{\lambda_3}{\lambda_1} \quad (11)$$

$$\text{diag}(\eta_1, \eta_2, \eta_3) = (\mathbf{V}^T \mathbf{R} \mathbf{U}) \cdot \text{diag}(\eta_1, \eta_2, \eta_3) \cdot (\mathbf{V}^T \mathbf{R} \mathbf{U})^T \quad (12)$$

Hence,  $\mathbf{W} = \mathbf{V}^T \mathbf{R} \mathbf{U}$  with  $\det(\mathbf{W}) = 1$  is made up of eigenvectors of non-singular diagonal matrix. And we can determine all the possible forms of  $\mathbf{W}$  as follows

$$\begin{aligned} \mathbf{W}_1 &= \text{diag}(1, 1, 1) & \mathbf{W}_2 &= \text{diag}(1, -1, -1) \\ \mathbf{W}_3 &= \text{diag}(-1, 1, -1) & \mathbf{W}_4 &= \text{diag}(-1, -1, 1) \end{aligned} \quad (13)$$

Thus, the rotation matrix  $\mathbf{R}$  can be solved as  $\mathbf{R} = \mathbf{V} \mathbf{W} \mathbf{U}^T$ , and we can obtain the position vector  $\mathbf{b}$  of the center of circle  $\mathbf{C}$  and the norm vector  $\mathbf{n}$  of circle plane in the camera frame from Eqs. (8)–(13) as

$$\begin{aligned} \mathbf{b} = \mathbf{t} &= -\frac{1}{s_3} \mathbf{R} \cdot (s_1, s_2, -1)^T = -\frac{1}{s_3} \mathbf{V} \mathbf{W}_k (\mathbf{U}^T \cdot (s_1, s_2, -1)^T) \\ &= r \sqrt{\frac{\lambda_1^2}{\lambda_2 \lambda_3}} [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3] \mathbf{W}_k \\ &\quad \cdot \begin{bmatrix} 0 & -\frac{\lambda_3}{\lambda_1} \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_2 - \lambda_3}} & \frac{\lambda_2}{\lambda_1} \sqrt{\frac{\lambda_3 - \lambda_1}{\lambda_3 - \lambda_2}} \end{bmatrix}^T \\ &= r \left( \pm \sqrt{\frac{-\lambda_3(\lambda_2 - \lambda_1)}{\lambda_2(\lambda_2 - \lambda_3)}} \mathbf{v}_2 \pm \sqrt{\frac{\lambda_2(\lambda_3 - \lambda_1)}{\lambda_3(\lambda_3 - \lambda_2)}} \mathbf{v}_3 \right) \end{aligned} \quad (14)$$

$$\begin{aligned} \mathbf{n} = \mathbf{r}_3 &= \mathbf{R} \mathbf{e}_3 = \mathbf{V} \mathbf{W}_k (\mathbf{U}^T \mathbf{e}_3) \\ &= \mathbf{V} \mathbf{W}_k \cdot \begin{bmatrix} 0 & \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_2 - \lambda_3}} & \sqrt{\frac{\lambda_3 - \lambda_1}{\lambda_3 - \lambda_2}} \end{bmatrix}^T \\ &= \pm \sqrt{\frac{\lambda_2 - \lambda_1}{\lambda_2 - \lambda_3}} \mathbf{v}_2 \pm \sqrt{\frac{\lambda_3 - \lambda_1}{\lambda_3 - \lambda_2}} \mathbf{v}_3 \end{aligned} \quad (15)$$

where  $k = 1, 2, 3, 4$ ,  $\mathbf{e}_3 = (0, 0, 1)^T$ , and  $\mathbf{V} = [\mathbf{v}_1 \ \mathbf{v}_2 \ \mathbf{v}_3]$  is made up of eigenvectors of  $\mathbf{Q}$ . Two solutions can be excluded when considering  $\mathbf{e}_3^T \mathbf{b} > 0$  in the actual situation.

In conclusion, we can easily achieve at most two solutions of  $\mathbf{b}$  and  $\mathbf{n}$  through diagonalizing  $\mathbf{Q}$  and applying Eqs. (14) and (15).

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