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Dynamic analysis of an autonomous chaotic system with cubic nonlinearity

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ABSTRACT

This work constructs an autonomous chaotic system with cubic nonlinearity. Some fundamental dynamical properties, such as equilibrium point, phase portrait, Lyapunov exponent, Kaplan–Yorke dimension, and bifurcation of the new system are investigated numerically and theoretically. Analyses show that this autonomous system holds rich dynamics and exhibits periodic, multi-periodic and chaos behavior with single-scroll, double-scroll and four-scroll.

1. Introduction

As a typical nonlinear phenomenon, chaos holds some special properties as sensitive to initial conditions, dense periodic orbits and topologically mixing. Chaotic behave exists widely in engineering, biology, economy and many other scientific disciplines [1–3].

Since the first chaotic attractor is reported by Lorenz [4], people focused their attention on generating chaotic behave in simple autonomous systems for the potential technological applications [5–15]. In this process, two stationary research directions are taking shape. The first one is to generate hyperchaotic system with at least two positive Lyapunov exponents [16,17]. Another important research direction is to construct chaotic attractors with complicated topological structures, such as ring-scroll and multi-scroll chaotic attractors. Recently, some smooth systems with four-scroll attractors have been presented [17–22], which holds the following common characteristics: (1) The nonlinearities are continuous or smooth functions, including cubic cross-product terms, quadratic cross-product terms, and so on. (2) These systems have five unstable equilibrium points: four nonzero equilibrium points and one zero equilibrium point. Moreover, each scroll wanders around the nonzero equilibrium point yet not the origin.

It's important and challenging to construct smooth autonomous systems with multi-scroll attractors for the simple circuit implementation and complicated topological structures. However, there are still very rare works on generating a multi-scroll attractor in three-dimensional smooth system.

In this work, we introduce a three-dimensional chaotic system with cubic nonlinearity. Some fundamental dynamical properties, such as equilibrium point, phase portrait, Lyapunov exponent, Kaplan–Yorke dimension, Poincare mapping, frequency spectra and bifurcation of the new system are investigated numerically and theoretically. Analyses show that this autonomous system holds rich dynamics and exhibits periodic, multi-periodic and chaos behavior with single-scroll, double-scroll and four-scroll.

2. Description of the new chaotic system

The three-dimensional chaotic system considered here is depicted as

$$\begin{cases} \dot{x}_1 = -ax_1 + bx_2x_3 \\ \dot{x}_2 = -c{x_2}^3 + dx_1x_3 \\ \dot{x}_3 = ex_3 - fx_1x_2 \end{cases} \tag{1}$$

System (1) consists of two quadratic cross-product terms and a cubic term, and the parameters a,b,c,d,e,f are the positive constant.

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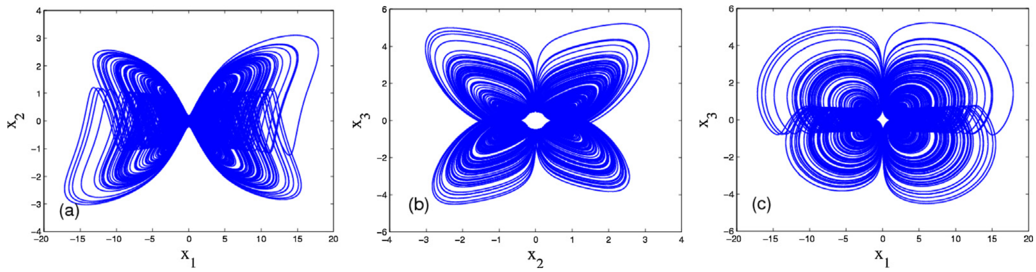

Fig. 1. Four-scroll attractor of system (1). (a) x_1–x_2 plane;(b) x_2–x_3 plane; (c) x_1–x_3 plane.

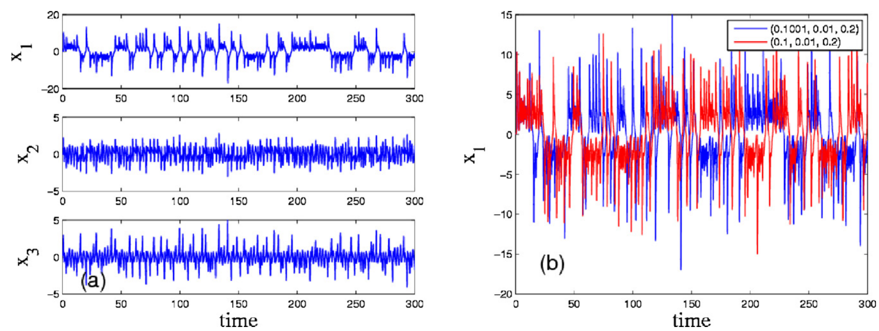

Fig. 2. (a) Time series of x_1, x_2 and x_3; (b) sensitive dependence on initial conditions.

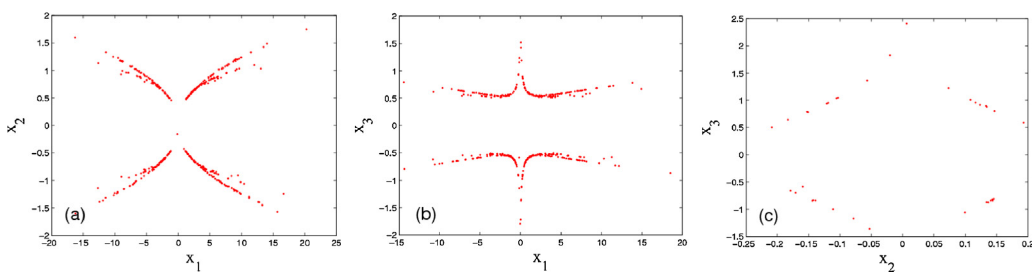

Fig. 3. Poincare maps on the plane of (a) $x_3 = 0$; (b) $x_2 = 0$ and (c) $x_1 = 0$.

2.1. Dissipativity and existence of attractor

To ensure that system (1) is chaotic, one should first consider the general condition of dissipativity

$$\nabla V = \frac{\partial \dot{x}_1}{x_1} + \frac{\partial \dot{x}_2}{x_2} + \frac{\partial \dot{x}_3}{x_3} = -a - 3cx_2^2 + e \quad (2)$$

So, system (1) would be dissipative when $-a - 3cx_2^2 + e < 0$, and will converges to a subset of measure zero volume according to $dV/dt = e^{-a-3cx_2^2+e}$. This means that the volume will become $V(0)e^{-(a+b)t}$ at time t through the flow generated by the system for an initial volume V_0. Therefore, there should exist an attractor in system (1).

2.2. Equilibrium and stability

Taking the condition $\dot{x}_1 = 0$, $\dot{x}_2 = 0$, $\dot{x}_3 = 0$ and set $a = 2$, $b = 10$, $c = 6$, $d = 3$, $e = 3$, $f = 1$, one gets five equilibrium points of system (1):
$P_0(0, 0, 0)$, $P_1(1.8974, 0.7746, 0.4899)$, $P_2(-1.8974, 0.7746, -0.4899)$, $P_3(-1.8974, -0.7746, 0.4899)$, $P_4(1.8974, -0.7746, -0.4899)$.

And the corresponding characteristic roots are
P_0: $\lambda_1 = -2$, $\lambda_2 = 0$, $\lambda_3 = 3$.
P_1 to P_4: $\lambda_1 = -11.1421$, $\lambda_2 = 0.6711 + 2.7026i$, $\lambda_3 = 0.6711 - 2.7026i$.

Clearly, all the equilibrium points are unstable with stable manifold and unstable manifold.

2.3. Phase portrait and chaotic properties

When selecting $a = 2$, $b = 10$, $c = 6$, $d = 3$, $e = 3$, $f = 1$, the Lyapunov exponents of system (1) are calculated as 1.0597, $-5.53E-4$, -18.29396. The corresponding Kaplan–Yorke dimension of the system is $D_{KY} = 2 + (1.0597 - 5.53E\text{-}4)/18.29396 = 2.0579$. Therefore, system (1) is indeed chaotic with fractional Kaplan–Yorke dimension. The corresponding chaotic phase diagrams are depicted in Fig. 1. It appears from Fig. 1 that the reported system displays complicated dynamical behaviors.

For a differential dynamical system, we call it's sensitive to initial conditions if a tiny change in the initial trajectory causes a big change in the final trajectory. For a chaotic system, sensitivity to initial conditions is one pervasive feature and is the main way in which one can recognize it from other system. The time series of x_1, x_2 and x_3 with $a = 2$, $b = 10$, $c = 6$, $d = 3$, $e = 3$, $f = 1$ and the initial condition (0.1, 0.001, 0.2) are shown in Fig. 2(a). Two orbits of x_1 with initial conditions (0.1001, 0.001, 0.2) and (0.1, 0.001, 0.2) are plotted in Fig. 2(b). It's known that the two time series begin with a tiny difference, but after a few iterations, the difference between them grows rapidly. This implies that the dynamical system (1) is sensitive to initial conditions.