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Coherently coupled spatial soliton pairs in biased photorefractive crystals with both the linear and quadratic electro-optic effects

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ABSTRACT

A study of coherently coupled spatial soliton pair propagating along the same line in biased photorefractive crystals where the changes of refractive index are governed by both the linear and quadratic electro-optic effects simultaneously is presented. It is shown that coherently coupled bright–bright, dark–dark and gray–gray spatial soliton pairs can be supported in steady-state regime under appropriate conditions. Moreover, the effects of three physical factors, i.e., the intensity ratio and the initial phase difference between two incident coherent beams, and various external bias field on the existence conditions, properties of these coherently coupled soliton pairs have been discussed in detail.

1. Introduction

Because of their unique features of formation at low laser power and important potential applications [1–3], the attention to photorefractive (PR) spatial solitons has transferred from the formation and properties to the interactions between spatial solitons. Among soliton interactions, soliton pairing has always been an appealing issue. When two incident beams propagate in the PR materials, these two beams trap mutually under the influence of the nonlinear refractive-index modulation and then each of them propagates undistorted, i.e., coupled spatial soliton pairs form. It needs to be emphasized that every component of coupled spatial soliton pairs depends on each other and each beam alone cannot survive as a soliton if the other beam is absent. Incoherently coupled screening spatial soliton pairs was firstly predicted by Christodoulides et al. [4] and experimentally observed by Chen et al. [5–7]. Soon later, Hou et al. presented that incoherently coupled multiple bright, dark and dark–bright hybrid screening-photovoltaic soliton families can be supported in biased photovoltaic PR materials [8–11]. In addition, the interactions between other types of PR spatial solitons resulting only from the linear or quadratic electro-optic (EO) effect have also been widely studied [12–16]. However, a number of types of new EO materials that have large EO effect including both the linear and quadratic EO effects near the phase-transition temperature have been found, such as ferroelectric $KTa_xNb_{1-x}O_3$ (KTN) crystals [17], $Pb(Zn_{1/3}Nb_{2/3})O_3$-$PbTiO_3$ (PZN-xPT) single crystals [18], $Pb(Mg_{1/3}Nb_{2/3})O_3]_{(1-x)}$-$(PbTiO_3)_x$ (PMN-xPT) [19] single crystals and so on. Recently, we have demonstrated that spatial solitons and incoherently coupled soliton families can exist in biased PR crystals involving both the linear and quadratic EO effect in steady-state regime [20,21]. The incoherent interactions of spatial solitons have already been widely concerned in recent two decades, whereas the researches regarding the coherent interactions are relatively few. Even in the research field of coherent interactions, much attention has been paid to the coherent coupling of beams that propagate non-collinearly in PR materials, and most works are based on the experimental methods. So far, the issue of coherent coupling of solitons propagating collinearly in PR materials has not been reported yet.

In this paper, we investigate theoretically coherently coupled bright–bright, dark–dark and gray–gray soliton pairs propagating collinearly in biased PR crystals with both the linear and quadratic electro-optic effects. Our results show that these soliton pairs owe their existence to both the linear and quadratic EO effects in the meantime. Moreover, the effects of the intensity ratio, the initial phase difference between two coherent incident beams, and various external bias field on the existence conditions and characteristics of these coherently coupled soliton pair will be discussed in detail.

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2. Theoretical model

We consider two mutually coherent incident beams that propagate collinearly along the z axis in biased PR crystal with both the linear and quadratic EO effects which is put with its principal axes aligned with the x, y and z directions of the system. The polarization of coherent incident beams and an external bias electric field are both assumed to be parallel to the x-axis. For simplify, only the x-axis diffraction is taken into account. The optical fields of two coherent incident beams are expressed as the slowly varying envelopes, i.e., $\vec{E}_k(x,z) = \hat{x}\varphi_k(x,z)\exp[i(kz+\phi_k)]$, ϕ_k represent the initial phases of two coherent incident beams, $k=1$, 2. $k = k_0 n_e = (2\pi/\lambda_0)\, n_e$ with n_e being the unperturbed index of refraction and λ_0 the free-space wavelength. Under these conditions, these two coherent beams satisfy the following equations [1,2]:

$$\left(i\frac{\partial}{\partial z} + \frac{1}{2k}\frac{\partial^2}{\partial x^2} + \frac{k}{n_e}\Delta n\right)\varphi_k(x,z) = 0 \quad k = 1, 2 \tag{1}$$

where the change of nonlinear refractive index Δn is governed by

$$\Delta n = -\frac{n_e^3 r_{33} E_{sc}}{2} - \frac{n_e^3 g_{\text{eff}}\varepsilon_0^2(\varepsilon_r - 1)^2 E_{sc}^2}{2} \tag{2}$$

and r_{33} and g_{eff} are linear and the effective quadratic EO coefficient of PR crystal, respectively. ε_0 and ε_r are the vacuum and relative dielectric constants, respectively. E_{sc} is the space-charge field in the material. Under a strong bias field condition, the drift effect will be dominant, thus E_{sc} can be approximately expressed as [22]

$$E_{sc} = E_0\frac{I_\infty + I_b + I_d}{I + I_b + I_d} \tag{3}$$

where I_d and I_b are the intensities of so-called dark irradiance and background beam, respectively. $I = I(x,z) = (n_e/2\eta_0)\left(\left|\sum_{k=1}^{2}\varphi_k\right|^2\right)$ is the total intensity of two incident beams with $\eta_0 = (\mu_0/\varepsilon_0)^{1/2}$. $E_0 = E_{sc}(x \to \pm\infty, z)$ and $I_\infty = I(x \to \pm\infty, z)$ represent the space charge field and the total intensity for $x \to \pm\infty$, respectively. In most cases, E_0 is approximately equal to $\pm V/W$, where V is the applied external voltage and W is the x-width of the PR crystal.

The normalized evolution equations can now be obtained by submitting Eqs. (2) and (3) into (1). For convenience, we adopt the following dimensionless parameters: $\xi = z/kx_0^2$, $s = x/x_0$, $\varphi_k = \left[2\eta_0(I_b + I_d)/n_e\right]^{1/2}U_k$, $k = 1, 2$. x_0 is an arbitrary spatial width. Under these conditions, the envelopes U_k are found to obey

$$i\frac{\partial U_k}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_k}{\partial s^2} - \frac{\beta_1(1+\rho)}{1 + \left|\sum_{k=1}^{2}U_k\right|^2}U_k - \frac{\beta_2(1+\rho)^2}{\left(1 + \left|\sum_{k=1}^{2}U_k\right|^2\right)^2}U_k = 0 \quad k = 1, 2 \tag{4}$$

where $\beta_1 = (k_0x_0)^2 n_e^4 r_{33}E_0/2$, $\beta_2 = (k_0x_0)^2 n_e^4 g_{\text{eff}}\varepsilon_0^2(\varepsilon_r - 1)^2E_0^2/2$, $\rho = I_\infty/(I_d + I_b)$. For simplicity, any loss effects have been neglected in our analysis.

3. Results and discussions

In what follows, we will solve Eq. (4) and present coherently coupled bright–bright, dark–dark and gray–gray soliton pair solutions in the steady-state regime. The existence conditions and properties of these soliton pair will be discussed in detail.

3.1. Coherently coupled bright–bright soliton pair solution

For bright–bright soliton pair, $I(0) = I_{\max}$, $\rho = I_\infty/(I_b + I_d) = 0$, and then Eq. (4) can be reduced to

$$i\frac{\partial U_k}{\partial \xi} + \frac{1}{2}\frac{\partial^2 U_k}{\partial s^2} - \frac{\beta_1}{1 + \left|\sum_{k=1}^{2}U_k\right|^2}U_k - \frac{\beta_2}{\left(1 + \left|\sum_{k=1}^{2}U_k\right|^2\right)^2}U_k = 0 \quad k = 1, 2 \tag{5}$$

To obtain the coherently coupled bright–bright soliton pair solution, we assume $U_k(s,\xi) = r_k^{1/2}y(s)\exp\left[i\left(\nu\xi + \phi_k\right)\right]$ $(k = 1, 2)$ where ν represents a nonlinear shift of the propagation constant, ϕ_k denotes the initial phase of two coherent beams. r_k are defined as $r_k = I_{k\max}/(I_b + I_d) = I_k(0)/(I_b + I_d)$. $y(s)$ is a normalized real function and satisfies $y(0) = 1$, $y'(0) = 0$, $y(s \to \pm\infty) = 0$ and all the derivatives of $y(s)$ are zeros when $s \to \pm\infty$. Substituting the expressions U_k into Eq. (5), we get

$$y'' - 2\nu y - 2\beta_1\frac{y}{1 + ry^2} - 2\beta_2\frac{y}{\left(1 + ry^2\right)^2} = 0 \tag{6}$$

where $r = r_1 + r_2 + 2(r_1r_2)^{1/2}\cos\Delta\phi$ with $\Delta\phi = \phi_1 - \phi_2$ being the phase difference between two coherent beams. For integration of Eq. (6) twice with employing y-boundary conditions, we find that

$$\nu = -\frac{\beta_1}{r}\ln(1 + r) - \frac{\beta_2}{1 + r} \tag{7}$$

$$s = \pm\int_y^1 \frac{d\tilde{y}}{\left\{\frac{2\beta_1}{r}\left[\ln\left(1 + r\tilde{y}^2\right) - \tilde{y}^2\ln(1 + r)\right] + \frac{2\beta_2}{1+r}\frac{r\tilde{y}^2(1-\tilde{y}^2)}{1+r\tilde{y}^2}\right\}^{1/2}} \tag{8}$$

By the numerical integration procedures $y(s)$ can be then obtained and the existence condition of coherently coupled bright–bright soliton pair is $\beta_1 > -\beta_2/6$ when $\beta_2 > 0$ or $\beta_1 > -2\beta_2$ when $\beta_2 < 0$ [20]. Quite different from the cases investigated previously, bright–bright soliton pair in our analysis can also exist when $\beta_1\beta_2 < 0$ so long as $\beta_1 > -\beta_2/6$ when $\beta_2 > 0$ or $\beta_1 > -2\beta_2$ when $\beta_2 < 0$ is satisfied. This can occur because of the interaction between the linear and quadratic EO effect where photorefractive effect can be enhanced or weakened even counteracted. By altering the polarity of E_0, we can make the sign of β_1 and β_2 different since the sign of the linear EO term $(\beta_1 \propto E_0)$ is changed whereas the quadratic EO term $(\beta_2 \propto E_0^2)$ is not influenced. Further, the two components of bright–bright soliton pair can also be obtained by the expressions of $U_k(s,\xi) = r_k^{1/2}y(s)\exp\left[i\left(\nu\xi + \phi_k\right)\right]$ $(k = 1, 2)$.

To illustrate our results, we take PMN-0.33PT single crystal which shows maximal transparency, very good optical clarity and low propagation loss for example. The parameters of PMN-0.33 PT are $n_e = 2.562$, $g = g_{\text{eff}}\varepsilon_0^2(\varepsilon_r - 1)^2 = 1.36 \times 10^{-16}\,\text{m}^2/\text{V}^2$, $r_{33} = 182 \times 10^{-12}\,\text{m/V}$ [23–27]. Other parameters are adopted as $\lambda_0 = 632.8\,\text{nm}$, $x_0 = 40\,\mu\text{m}$, $r_1 = 5$ and $\rho_1 = 5$. Fig. 1 depicts the intensity profiles of soliton components of coherently coupled bright–bright soliton pair components for (a) $\Delta\phi = \pi/4$, (b) $\Delta\phi = 3\pi/4$ when $r_2/r_1 = 1.6$, and $E_0 = 3 \times 10^5\,\text{V/m}$. Based on the above parameters, $\beta_1 = 185.54$ and $\beta_2 = 41.68$. The FWHMs of bright–bright soliton pair are 10.07 μm and 7.03 μm, respectively. Obviously, the phase difference $\Delta\phi$ has an impact on the widths of soliton pair. Besides the phase difference $\Delta\phi$, for coherently coupled soliton pair which propagates collinearly, the soliton pair