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A novel weak signal detection method for Linear Frequency Modulation signal based on bistable system and Fractional Fourier Transform

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ABSTRACT

Fractional Fourier Transform (FRFT) is regarded as an effective method for detection of Linear Frequency Modulation (LFM) signals in recent years. However, the performance of FRFT detection will deteriorate sharply in the weak noise scene. An alternative solution is the use of bistable system, which can generate stochastic resonance (SR), and is relatively easy to achieve. This paper proposes a novel LFM weak signal detection algorithm based on bistable system and FRFT, named BSFRFT. We use the SR effect of bistable system to amplify LFM signal, and then apply FRFT to the previous output. We also present an evaluation criteria to measure the ability of detection methods and to compare performance of FRFT and BSFRFT. Numerical simulations show that the algorithm is effective for two cases – the LFM signal covered by white noise or colored noise (including pink noise).

1. Introduction

Linear Frequency Modulation signal (LFM signal), which is always implemented as linear chirp signal, is widely used in sonar and radar systems. It could get larger compression ratio as well as having excellent range resolution and radial velocity resolution ration [1].

The typical LFM signal can be written as

$$S(t) = A_0 \cos(2\pi\mu t^2 + 2\pi f_0 t), \tag{1}$$

where A_0 is the signal amplitude, μ the rate of frequency increase or chirp rate, and f_0 the starting frequency.

There are many methods to estimate LFM signal due to their time-varing characteristic, such as short-time Fourier transform (STFT) [2], Wigner–Ville distribution (WVD) [3], Radon–Wigner transform (RWT) [4], Wigner–Hough transform (WHT) [5] and so on. However, the above approaches have some disadvantages. The STFT cannot achieve a satisfied time-frequency resolution due to the window function. The WVD suffers from the effect of cross-terms. Although some techniques have been proposed to suppress the cross-terms, its time–frequency resolution is then reduced and the computational complexity increased [6]. The RWT and WHT are both two-dimensional search algorithms, whose searching time is large and cross-terms interferences also exist [1].

Fractional Fourier Transform (FRFT) is a generalization of the conventional Fourier Transform. The LFM signal has the best characteristics of energy concentration in certain specific fractional Fourier domain [1], then FRFT provides a higher time–frequency resolution than STFT and it avoids the effect of cross-terms produced by the WVD [6]. Due to its orthonormal chirped basis, the LFM signal fits well in the FRFT domain for detection and estimation [6]. However, through the theoretical analysis of FRFT detection performance, it is noted that although FRFT is a linear transformation, the output SNR relative to the input SNR has a threshold effect – when the input SNR is higher than the threshold, the output SNR is improved and when the input SNR is below the threshold, the output SNR is worsened [10]. In other words, when input SNR is not large enough, the FRFT detection result is not satisfactory.

Stochastic resonance (SR) is a phenomenon wherein the response of a nonlinear system to a weak periodic or aperiodic input signal is optimized by the presence of a particular level of noise [8], and broadly adopted to describe any phenomenon where the presence of noise in a nonlinear system is better for output signal quality than its absence [7]. Bistable system is a simple model used to generate SR and has received much research attention. It has two stable equilibrium states and can transition from one state to the other if it is given enough activation energy to penetrate the barrier.

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Then SR effect of bistable system has simple physical explanation, that is, with the help of random forces, a particle makes occasional transitions from an equilibrium state over the barrier in the center and as the input noise variance is increased, the rate at which such jumps will occur increases, but once noise variance is large enough that the barrier becomes easy to surmount, the rate grows more slowly as the noise is further increased [9].

In this paper, we will use bistable system to amplify the input LFM signal covered by noise with the SR effect and then apply FRFT to detecting the output of bistable system.

2. LFM signal detection based on FRFT

2.1. Basis of FRFT

The FRFT of signal $x(t)$ is represented as

$$X_p(u) = F^p[x(t)] = \int_{-\infty}^{\infty} x(t)K_\alpha(t,u)dt, \tag{2}$$

where p is the fractional order, $\alpha = \frac{p\pi}{2}$, $F^p[\cdot]$ denotes the FRFT operator, and $K_\alpha(t,u)$ is the kernel of the FRFT:

$$K_\alpha(t,u) = A_\alpha \exp(j\pi(u^2 \cot\alpha - 2utcsc\alpha + t^2 \cot\alpha)) \tag{3}$$

with $A_\alpha = \sqrt{1 - j\cot\alpha}$. $K_\alpha(t,u)$ has the following properties:

$$K_{-\alpha}(t,u) = K_\alpha^*(t,u), \tag{4}$$

$$\int_{-\infty}^{\infty} K_\alpha(t,u)K_\alpha^*(t,u\prime) = \delta(u - u\prime). \tag{5}$$

Hence, the inverse FRFT is

$$x(t) = F^{-p}[X_p(u)] = \int_{-\infty}^{\infty} X_p(u)K_{-\alpha}(t,u)du. \tag{6}$$

Eq. (6) indicates that signal $x(t)$ can be interpreted as a decomposition to a basis formed by the orthonormal LFM functions in the u domain, and the u domain is usually called the fractional Fourier domain, in which the time and frequency domains are its special cases [11].

2.2. Discrete FRFT algorithm

Ozaktas gave an effective DFRFT algorithm [12]:

$$X_p(u) = \frac{A_\alpha}{2F}\sum_{n=-N}^{N} \exp(j\pi u^2 \cot\alpha)\exp\left(\frac{j\pi n^2 \cot\alpha}{(2F)^2} - \frac{j2\pi uncsc\alpha}{2F}\right) s\left(\frac{n}{2F}\right), \tag{7}$$

where p, α, A_α are defined as in Section 2.1, F is the highest frequency of signal $s(t)$, n is the sampling point for t, $N = F^2$, and $s(t)$ is assumed to be zero outside $\left[-\frac{F}{2}, \frac{F}{2}\right]$.

Go on to discrete u, we get that:

$$X_p\left(\frac{m}{2F}\right) = \frac{A_\alpha}{2F}\sum_{n=-N}^{N} \exp\left(\frac{j\pi m^2 \cot\alpha}{(2F)^2} + \frac{j\pi n^2 \cot\alpha}{(2F)^2} - \frac{j2\pi mncsc\alpha}{(2F)^2}\right) s\left(\frac{n}{2F}\right), \tag{8}$$

where m is the sampling point for u.

After some algebraic manipulations, we can rewrite (8) as the following form:

$$X_p\left(\frac{m}{2F}\right) = \frac{A_\alpha}{2F}\exp\left(j\pi(\cot\alpha - csc\alpha)\left(\frac{m}{2F}\right)^2\right)\sum_{n=-N}^{N} \exp\left(j\pi csc\alpha\left(\frac{m-n}{2F}\right)^2\right) \exp\left(j\pi(\cot\alpha - csc\alpha)\left(\frac{n}{2F}\right)^2\right) s\left(\frac{n}{2F}\right). \tag{9}$$

It can be recognized that the summation is the convolution of $\exp\left(j\pi csc\alpha\left(\frac{n}{2F}\right)^2\right)$ and the chirp-modulated function $s(\cdot)$. The

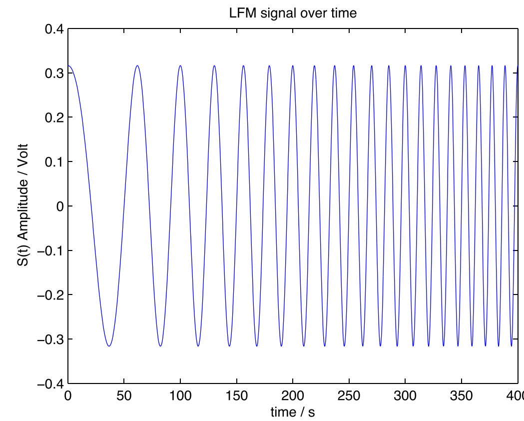

Fig. 1. LFM signal.

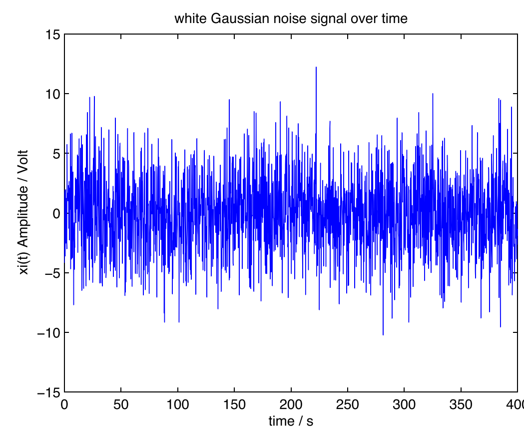

Fig. 2. Gaussian white noise.

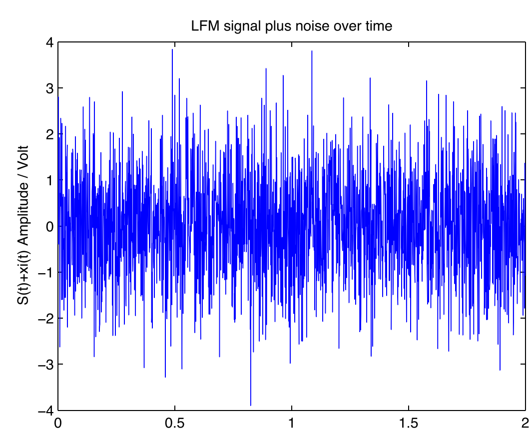

Fig. 3. LFM signal plus Gaussian white noise.