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Computer simulation of Talbot phenomenon using the Fresnel integrals approach

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ABSTRACT

Talbot phenomenon of the finiteness grating is studied both theoretically and experimentally. The simulations can be performed in any PC using a MATLAB program developed by the author. The diffractions of finiteness periodic square aperture arrays in Fresnel fields are analyzed according to the scalar diffraction theory. The additional intensity maxima in Talbot images of the finiteness gratings are theoretically predicted to appear. The square aperture arrays are produced and the self-images of the gratings are measured successfully in the experiment. A comparative analysis of theoretical results with experimental ones is illustrated.

1. Introduction

The diffraction phenomenon is of the utmost importance in the theory of optical systems [1–4]. Most of the problems are not amenable to an exact solution in diffraction optics because of its complexity. Therefore, numerical methods facilitate the viable alternatives to investigate relatively simpler cases. Two types of diffraction are distinguished, depending upon the distance between the source and the screen: Fraunhofer diffraction or far-field diffraction at large distances and Fresnel diffraction or near-field diffraction at close distances [5]. Due to the finite distance between the initial plane and the observation plane, the optical wavefronts are not planar, in the Fresnel zone. The diffraction integral becomes more complex to find plausible solutions compared to those in Fraunhofer diffraction [4–6]. For Fresnel diffraction from single apertures (rectangular or circular) two-dimensional (2D) diffraction integral [5] can sometimes be separated into two separate functions in Cartesian or polar coordinates [7]. For example, the solution of Fresnel diffraction from rectangular aperture is usually derived in terms of certain non-analytic integrals known as Fresnel integrals, involving arguments in the Cartesian coordinates.

Talbot phenomenon is a recurrent self-imaging phenomenon in the near-field diffraction of plane waves from a grating [8–10], and the Talbot distance depends on the parameters of the grating ($z_T = 2D^2/\lambda$, where D is grating period and λ is the probe wavelength). In the past decade Talbot phenomenon has attracted a lot of attention, owing to its potential applications in image preprocessing and synthesis [11,12], photolithography [13], spectrometry [14], optical sensing [15,16], and elsewhere. New computers and powerful software has permitted the numerical computation of Talbot images [17,18], using 2D fast Fourier transform (FFT). While 2D FFT methods are very versatile and powerful, using them relegates the entire computation process to the computer, and provides little insight into the computation process itself. No existing symmetry property of the aperture is used to simplify the calculation process, nor any attempt is made to effect a separation of the diffraction integral into functions involving one-dimensional (1D) variables. Moreover, it is difficult to predict how the complex and intricate diffraction image will change with a change of experimental parameters (distance, period of the grating or its aperture). For these reasons, present paper shows the results of simulation of Talbot image of finiteness grating, using the Fresnel integrals approach instead of the more common FFT-based methods. Using the method, one can simulate the Fresnel diffraction pattern from a gating of any period in any experimental configuration by using a common PC and the well-known software MATLAB.

The layout of the paper is as follows: Section 2 shows the detailed underlying theory of the iterative Fresnel algorithm, details of simulation algorithm and results are described in Section 3, experimental results are shown in Section 4, finally some concluding remarks are made in Section 5.

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2. Theory

Light diffraction by a single aperture is discussed and analyzed in many textbooks [4–6]. Fig. 1 shows the basis of calculation of Fresnel diffraction from a single aperture.

Square aperture of dimensions $a \times a$ is illuminated with plane light beam of wavelength λ, and the diffracted light is observed on the screen located on a distance z. For convenience, the coordinate systems on the aperture and on the image planes are chosen to be centered on the optical axis passing through the center of the aperture and normal to it, and are denoted by (ξ, η) and (x, y) axes, respectively. The Huygens–Fresnel principle is then invoked to calculate the total electric field at any given point of the image plane (x, y) by summing up the contributions of all the elementary waves emitted by different areas inside the aperture, taking into account both amplitude and phase in the process.

In free-space light propagation at a distance z from the aperture the field amplitude becomes [4]:

$$U(x, y; z) = -jU_0 e^{j\frac{2\pi z}{\lambda}} \mathcal{E}(x)\mathcal{E}(y), \quad (1)$$

where U_0 is the amplitude of the incoming wave, and $\mathcal{E}(x)$ and $\mathcal{E}(y)$ are the Fresnel exponential integrals:

$$\mathcal{E}(w) = \int_{-\frac{a}{2\sqrt{\lambda z}}}^{\frac{a}{2\sqrt{\lambda z}}} e^{j\pi(w-\omega)^2} d\omega. \quad (2)$$

The irradiance at observation plane is given by the square of absolute value [4]:

$$I(x, y; z) = I_0 \left(\mathcal{E}(x)\bar{\mathcal{E}}(x)\right) \left(\mathcal{E}(y)\bar{\mathcal{E}}(y)\right). \quad (3)$$

Here I_0 is the unobstructed intensity corresponding to U_0 ($I_0 = \left|U_0\right|^2$).

Let us consider the case of a $N \times N$ apertures, with a filling factor $f = a/D$ where D is the period of the structure. The structure is centered on the (ξ, η) system, i.e. the origin the coordinate system is located at the exact center of the structure. The total amplitude distribution of electric field over the structure is given by:

$$U_S(\xi, \eta; 0) = U_0 \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} 1\left(\frac{\xi + D}{a}\left(n - \frac{N+1}{2}\right)\right) \times 1\left(\frac{\eta + D}{a}\left(m - \frac{N+1}{2}\right)\right), \quad (4)$$

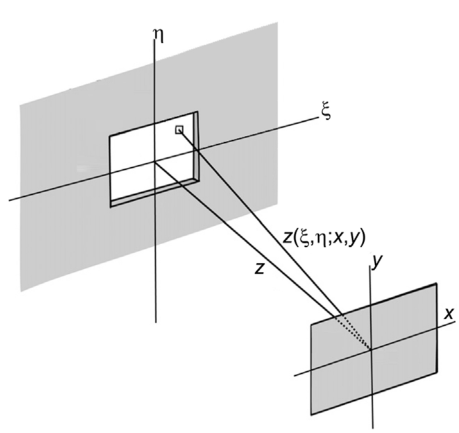

Fig. 1. Basic configuration of Fresnel diffraction from a single square aperture [5].

where $1(w)$ is the rectangular function [19]:

$$1(w) = \begin{cases} 0 & |w| > \frac{1}{2} \\ \frac{1}{2} & |w| = \frac{1}{2} \\ 1 & |w| < \frac{1}{2} \end{cases} \quad (5)$$

The total amplitude distribution of electric field at a distance z from the grating becomes:

$$U(x, y; z) = -jU_0 e^{j\frac{2\pi z}{\lambda}} \sum_{n=0}^{N-1}\sum_{m=0}^{N-1} \int_{-a_n}^{a_n} e^{j\pi(x-\xi)^2} d\xi \int_{-a_m}^{a_m} e^{j\pi(y-\eta)^2} d\eta, \quad (6)$$

where a_n and a_m are the single aperture shifts:

$$\pm a_i = \frac{1}{\sqrt{\lambda z}}\left(\pm\frac{a}{2} - \left(i - \frac{N-1}{2}\right)D\right). \quad (7)$$

For further consideration it is convenient to express (6) in terms of Fresnel exponential integral (2):

$$U(x, y; z) = -jU_0 e^{j\frac{2\pi z}{\lambda}} \sum_{n=0}^{N-1} \mathcal{E}\left(x - \left(n - \frac{N-1}{2}\right)D\right) \times \sum_{m=0}^{N-1} \mathcal{E}\left(y - \left(m - \frac{N-1}{2}\right)D\right). \quad (8)$$

As one can see from Eqs. (6) and (8) in the case of $a = D$ a single aperture of width ND is realized.

It is apparent that the computation of amplitude distribution (8) or intensity distribution (3) requires the evaluation of 2 Fresnel exponential integrals, which correspond to the 4 edges of the single aperture system, and the evaluation of $2N$ shifts (7), corresponding to the $N \times N$ apertures system. One should note, that MATLAB uses the cosine and sine Fresnel integrals, which form the real parts (cosine) and the imaginary parts (sine) of the amplitude. Euler's formula $\exp(j\varphi) = \cos(\varphi) + j\sin(\varphi)$ is used to calculate the complex amplitude distribution both single aperture (1) and $N \times N$ apertures system (8).

The calculation of the intensity distribution (8) may be done using Fresnel cosine and sine integrals for arbitrary argument value. MATLAB 6.5 has no built-in functions available to calculate the Fresnel integrals, but Fresnel cosine and sine integrals can be invoked by typing `mfun('FresnelS', phase)` and `mfun('FresnelS', phase)`, using Symbolic Math Toolbox of MATLAB. These functions return single values of Fresnel integrals if `phase` is a single valued variable. To generate Fresnel integrals from `Sphase` to `Ephase` with a step of `step` one should type `u=mfun('FresnelC', Sphase: step: Ephase)` for cosine and `u=mfun('FresnelS', Sphase: step: Ephase)` for sine, which were used in simulations. The complete MATLAB program is given in Appendix A. The initial parameters of the program are the length of the transparent part a (in mm), the period of the structure D (in mm), the aperture-image plane distance z (in mm), the step size or resolution s (in mm), the wavelength l (in nm), the image area b (in mm) and the number of apertures N.

3. Simulation results

Using the program, depicted in Appendix A, the values of the required inputs (aperture parameters, step size, the aperture-image plane distance, the light wavelength, the image area and the number of apertures) are input via a graphic user interface into the