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Scattering of a plane wave by a cylindrical parabolic perfectly electric conducting reflector

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ABSTRACT

Scattering of a plane wave incident on a cylindrical parabolic PEC surface with an arbitrary angle is obtained by the stationary phase point and edge point methods for the reflected and diffracted fields, respectively. These fields are plotted numerically.

1. Introduction

Scattering of high-frequency fields by conducting surfaces having cylindrical shapes have been under research [1–4]. Cylindrical reflector antennas have been studied for a long time.

Asymptotic evaluation of the reflected field was carried out for plane and cylindrical waves being incident on circular and parabolic cylinders [5]. Reflectors that may be considered as the most widely used high-gain antennas are used in long-distance communication systems and radar applications requiring high resolutions. Gains that are more than 30 dB can be achieved by reflectors but not by any other type of a single antenna [6]. The most common type of high-gain reflectors is the parabolic antenna. Circular cylindrical reflectors for scattering of waves are studied as the part of curved shapes [7]. Scattering of the waves created by a line source located at the focus of a cylindrical parabolic impedance surface is investigated as well [8]. Besides, studies in the scattering of inhomogenous waves, a part of our study in this paper, are keeping pace with the other classes of works in electromagnetic scattering phenomena [9–13]. Inhomogenous waves are the members of evanescent waves whose intensity decays with the distance in the way that is not in the same direction as the direction of propagation. They are studied by using the method of complex rays [14–16] that are calculated by the solution of the high frequency Luneberg–Kline expansion of the electromagnetic field having complex-valued amplitude and phase functions [17,18]. They are negligible in the far zone by knowing that they are exponentially small but are taken into consideration in the near field due to the contribution to the field in this region [17]. Scattering of evanescent waves by cylindrical structures is also studied [19].

In this study, scattering of a plane wave coming with an arbitrary complex angle to the cylindrical parabolic reflector is investigated.

A time factor of $\exp(j\omega t)$ is assumed and suppressed throughout the paper.

2. Theory

A cylindrical parabolic perfectly electric conducting (PEC) reflector whose surface equation is given in Eq. (1) is placed on a coordinate system symmetrical with respect to x-axis as shown in Fig. 1.

$$\rho' = \frac{f}{\cos^2(\varphi'/2)} \tag{1}$$

where φ' and ρ' are cylindrical coordinates and f is the focus of the parabola.

Normal vector to the surface is written as,

$$\vec{n} = -\cos\left(\frac{\varphi'}{2}\right)\hat{e}_\rho + \sin\left(\frac{\varphi'}{2}\right)\hat{e}_\varphi. \tag{2}$$

θ is found as,

$$\theta = \frac{\varphi'}{2}. \tag{3}$$

The scattering integral [20] of MTPO [21] for an impedance surface can be applied to a perfectly electric conducting (PEC) surface

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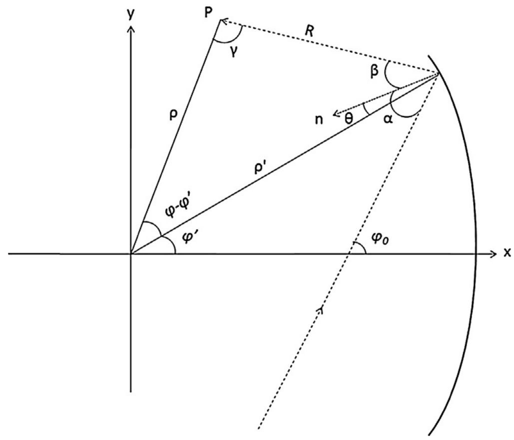

Fig. 1. Geometry of the cylindrical parabolic PEC reflector.

as,

$$E_s = \frac{k\ \exp\left(j\pi/4\right)}{2\pi}\int_{-\varphi_e}^{\varphi_e}\cos\left(\frac{\beta+\alpha}{2}\right)\frac{\exp\left[jk(\rho'\cos\left(\varphi'-\varphi_0\right)+R)\right]}{\sqrt{kR}}\frac{\rho'}{\cos\left(\varphi'/2\right)}d\varphi' \quad (4)$$

where

$$R = \rho\cos\gamma + \rho'\cos\left(\beta+\frac{\varphi'}{2}\right) \quad (5)$$

3. Asymptotic evaluation of the scattering integral

The phase function of the scattering integral is

$$\psi = \rho'\cos\left(\varphi'-\varphi_0\right)+R \quad (6)$$

And its first derivative is found to be

$$\frac{\partial\psi}{\partial\varphi'} = \rho'\frac{\sin\left(\varphi_0-\varphi'/2\right)}{\cos\left(\varphi'/2\right)} - \frac{\rho'}{\cos\left(\varphi'/2\right)}\sin\beta \quad (7)$$

By equating the first derivative to zero, we obtain the stationary phase point of β as

$$\beta_s = \alpha_s = \varphi_0 - \frac{\varphi_s}{2} \quad (8)$$

At the stationary point second derivative of the phase function is concluded as,

$$\psi_s'' = \frac{\rho_s}{\cos\left(\varphi_s/2\right)}\left[-\cos\left(\varphi_0-\frac{\varphi_s}{2}\right)+\frac{\rho_s\cos^2\left(\varphi_0-\varphi_s/2\right)}{l\cos\left(\varphi_s/2\right)}\right] \quad (9)$$

where φ_s, ρ_s and l are the stationary values of φ', ρ' and R, respectively. Reflected field can be written as,

$$E_r = f_s\exp\left(\left(-jk\left(\rho_s\cos\left(\varphi_s-\varphi_0\right)+l\right)\right)\right)\int_{-\infty}^{\infty}\exp\left\{-jk\frac{\rho_s\cos\left(\varphi_0-\varphi_s/2\right)}{2l\cos^2\left(\varphi_s/2\right)}\times\left[\rho_s\cos\left(\varphi_0-\frac{\varphi_s}{2}\right)-l\cos\left(\frac{\varphi_s}{2}\right)\right]\left(\varphi'-\varphi_s\right)^2\right\}d\varphi' \quad (10)$$

where

$$f_s = \frac{k\exp\left(j\pi/4\right)}{\sqrt{2\pi}}\cos\left(\frac{\left(\beta_s+\alpha_s\right)}{2}\right)\frac{\rho_s}{\sqrt{kl}\cos\left(\varphi_s/2\right)} \quad (11)$$

And finally, reflected field is reduced to,

$$E_r = \exp\left(-jk\left(\rho_s\cos\left(\varphi_s-\varphi_0\right)+l\right)\right)\sqrt{\frac{\rho_s\cos\left(\varphi_0-\varphi_s/2\right)}{\rho_s\cos\left(\varphi_0-\varphi_s/2\right)-l\cos\left(\varphi_s/2\right)}} \quad (12)$$

4. Diffracted fields

Diffracted field can be written as [8],

$$E_d \cong \frac{1}{jk}\left[\frac{f\left(\varphi_e\right)}{g'\left(\varphi_e\right)}\exp\left(-jkg\left(\varphi_e\right)\right)-\frac{f\left(-\varphi_e\right)}{g'\left(-\varphi_e\right)}\exp\left(-jkg\left(-\varphi_e\right)\right)\right] \quad (13)$$

where

$$f\left(\varphi_e\right) = \frac{k\ \exp\left(j\pi/4\right)}{\sqrt{2\pi}}\frac{\rho_e}{\sqrt{kR_e}\cos\left(\varphi_e/2\right)} \quad (14)$$

and

$$g\left(\varphi_e\right) = \rho_e\cos\left(\varphi_e-\varphi_0\right)+R_e \quad (15)$$

$$E_d = \frac{\exp\left(-j\pi/4\right)\exp\left(jkR_e\right)\exp\left(jk\rho_e\cos\left(\varphi_e-\varphi_0\right)\right)}{\sqrt{2k\pi R_e}}\left[\frac{\cos\left(\varphi_0-\varphi_e/2\right)}{\sin\left(\varphi_0-\varphi_e/2\right)-\sin\beta_{e1}}-\frac{\cos\left(\varphi_0+\varphi_e/2\right)}{\sin\left(\varphi_0+\varphi_e/2\right)-\sin\beta_{e2}}\right] \quad (16)$$

where

$$R_e^2 = \rho^2-\rho_e^2-2\rho\rho_e\cos\left(\varphi-\varphi_e\right) \quad (17)$$

For the upper edge the diffracted field can be rewritten as,

$$E_d = \frac{\exp\left(-j\pi/4\right)\exp\left(jkR_e\right)\exp\left(jk\rho_e\cos\left(\varphi_e-\varphi_0\right)\right)}{2\sqrt{2k\pi R_e}}\left[\frac{A}{\cos\left(\left(\varphi_0-\left(\varphi_e/2\right)+\beta_{e1}\right)/2\right)}+\frac{B}{\sin\left(\left(\varphi_0-\left(\varphi_e/2\right)-\beta_{e1}\right)/2\right)}\right] \quad (18)$$

where A and B are the undetermined coefficients. By the approach used in [22,23] if we let,

$$t_1 = -\sqrt{2kR_e}\cos\left(\frac{\left(\varphi_0-\left(\varphi_e/2\right)+\beta_{e1}\right)}{2}\right), \quad (19)$$

we obtain,

$$kR_e = 2kR_e\cos^2\left(\frac{\left(\varphi_0-\left(\varphi_e/2\right)+\beta_{e1}\right)}{2}\right)-kR_e\cos\left(\varphi_0-\left(\frac{\varphi_e}{2}\right)+\beta_{e1}\right). \quad (20)$$

By using the Fresnel function of

$$F[x] = \frac{\exp\left(j\pi/4\right)}{\sqrt{\pi}}\int_x^{\infty}\exp\left(-jy^2\right)dy, \quad (21)$$

reflected part of Ed can be written as,

$$E_{d1} = -A\,\mathrm{sign}\left(t_1\right)F\left[\left|t_1\right|\right]\cos\left(\varphi_0-\varphi_e/2\right)\exp\left(jkR_e\cos\left(\varphi_0-\frac{\varphi_e}{2}+\beta_{e1}\right)\right), \quad (22)$$