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Optimal control on decoherence for Markovian quantum system

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The optimal control function for an open system is designed by taking the expectation value of the state under control reaching the target state as the performance indicator. Utilizing Runge Kutta method, the open quantum system with a superconducting qubit is numerically investigated. The influences of parameters on the population transfer probability and control time are compared. The simulation experiments indicate that the state transfer and state maintenance is realized with a success rate of more than 98 percent, which validating the application of optimal control method in the state transfer and maintenance for open quantum system containing superconducting qubits. Moreover, proper increase of the proportional coefficients can accelerate the qubits flip and reduce the vibration frequency of control function.

1. Introduction

Superconducting devices such as Cooper pair boxes, Josephson junctions, and superconducting quantum interference devices (SQUIDs) have attracted much attention in the quantum information community [1,2]. Because they are relatively easy to scale up and have been demonstrated to have relatively long decoherence time [3], they have been considered as promising candidates for physical implementation of quantum computation. The superconducting qubit program based on Josephson junction therefore becomes the main research area for the moment and has shown immense application potential. Corresponding experimental programs related to the qubit have been carried out [4].

With the development of communication technology and quantum computation, the dissipative behavior in quantum physics is the most attractable part in modern physical quantum state engineering [5]. The quantum state, however, is very sensitive to the perturbation factors from environment. Owing to coupling quite a lot of environmental degree of freedom, the processing of decoherence issue becomes very prominent in superconducting quantum computation and even in almost all the solid-state quantum computation [6,7]. For one thing, the mechanism of decoherence is relatively complicated since much interaction exists in the solid, for instance, the solid-state qubits will inevitably interact with the phonon environment in the solid, and the decoherence originated by the phonons is a problem that must be known; For another, among the noises in various of solid components, 1/f noise and the flicker noise are most difficult to deal with and their physical mechanisms are still unsure. Therefore, it is great significant for such projects as realizing application of the quantum information techniques

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in quantum nanometer structure, precisely understanding decoherence in dissipative quantum systems, and achieving the quantum coherent manipulation as much as possible in specific coherent time.

Quantum information technologies are composed of quantum computation, quantum communication and quantum control [8]. Currently, quantum control mainly applies the classical theory and modern techniques to realize the manipulation of quantum states. It is relative easy for the closed system since it corresponds to the ideal system model, and the control of many quantum systems has been realized successfully. For the closed system, several strategies have emerged, eg., optimal control, control theory of bilinear systems, and Lypunov methods [9,10]. The theory of optimal control was introduced in the theory of automatic control in the 1960s for electrical engineering applications. In quantum chemical, quantum optimal control made great success. Recent work has optimized the control law for closed or open system population transfer by using a gradient ascent pulse engineering algorithm [11,12]. Optimal control has been exploited to control the quantum decoherence, where an optimal control law was designed to effectively suppress decoherence effects in Markovian open quantum systems.

Quantum computation requires precise and complete control on quantum systems. We design control laws for states transfer based on optimal control theory, and drive the system to the desired target state by adjusting control parameters. Taking the typical experimental data of decoherece of superconducting qubits as simulation parameters, we numerically investigate the free evolution of open quantum system, and the complete control on the states transfer between arbitrary two states under the designed control laws.

2. Model

Superconducting qubits usually can be divided into three types, charge qubits, phase qubits and flux qubits. Each of them has its own advantages. The charge qubits can be flexibly regulated by the external parameters such as bias magnetic flux and voltage, and this makes them suitable for operation qubits. The flux qubits have a longer decoherence time, so they are fit for storage qubits. Either way, different from the realistic atom, the Hamitonian of the superconducting qubit known as the artificial atom is ($k_B = \hbar = 1$):

$$\hat{H}_S = -\frac{1}{2}B_z\hat{\sigma}_z - \frac{1}{2}B_x\hat{\sigma}_x, \tag{1}$$

where σ_k with $k = x, y, z$ are the Pauli matrices. The energy bias B_z is controlled by the externally applied flux $\Phi(t)$. B_x is the tunneling amplitude of the qubit. For convenience, we discuss the optimal control of state transfer in the open quantum system containing superconducting charge qubits. The control method is easy to extend to other superconducting qubit system.

The process of designing practical superconducting charge qubits is introduced in Refs. [13,14]. Make $C_g(C_J)$ as gate (junction) capacitance, use superconducting quantum interferometer instead of the single Josephson junction, consider the voltage near the degeneracy point only has two charge states $n = 0$ and $n = 1$, then B_z and B_x of the charge qubits could be,

$$B_z = 4E_C\left(1 - 2n_g\right). \tag{2}$$

$$B_x = 2E_J^0\cos\left(\pi\frac{\Phi}{\Phi_0}\right). \tag{3}$$

$E_C = e^2/2(C_g + C_J)$ is the singlet electron charge energy, $n_g = C_gV_g/2e$ is dimensionless gate charge, demonstrating the influence of the gate voltage V_g. Near the degeneracy point, $n_g \approx 0.5$. E_J^0 is the Josephson energy of singlet Josephson junction, which is proportional to critical current I_C passed through Josephson junction. Φ is the magnetic flux in the superconducting circuit, Φ_0 is flux quantum. For charge qubit, we always have $E_C >> E_J^0$.

The control on quantum system is realized usually through interacting the externally applied optical field or electromagnetic field with the dynamical variables of the system, which is equivalent to introduce some Hamiltonian into the original Hamiltonian to change the energy of the system. Obviously, superconducting charge qubit can be easily controlled by external electromagnetic field: the z component B_z of effective magnetic field can be controlled through the gate voltage (effective gate charge n_g); the x component B_x of effective magnetic field can be controlled through changing external magnetic flux$\Phi(t)$.

The decoherence and relaxation of the superconducting qubit may be considered. Suppose the superconducting qubit interacts through $\sigma_k(k = x, y, z)$ operator with the environment. The $\sigma_- = (\sigma_x - i\sigma_y)/2$ coupling to the environment models the relaxation (and thus also decoherence) process of the superconducting qubit, while the σ_z coupling to the environment models the pure dephasing process of the superconducting qubit. Generally speaking, Markovian approximation is used under the assumption that the correlation time between the systems and environments is infinitely short. Neglecting the memory effect, the Lindblad master equation is built [15]:

$$\frac{d}{dt}\rho(t) = -i\left[H_S + H_C(t), \rho(t)\right] + \gamma_1 D[\sigma_-]\rho(t) + \gamma_\phi D[\sigma_z]\rho(t), \tag{4}$$

$$D[A_k]\rho(t) = \hat{A}_k(t)\hat{\rho}(t)\hat{A}_k^+(t) - \frac{1}{2}\left\{\hat{A}_k^+(t)\hat{A}_k(t), \hat{\rho}(t)\right\}, \tag{5}$$