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# A novel fractal method for fault diagnosis and signal measurements

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ABSTRACT

In this work, a wavelet fractal method is proposed by combining the fractal theory with a wavelet decomposition. The proposed method along with the correlation dimension acquired from the generic programming method is applied to characterize the experimental signals under three different conditions. Both methods show the obvious advantages in extracting characteristics from the signals under different conditions. By comparing the statistical data of such two methods, it can be concluded that the characteristics obtained from the wavelet fractal method have shown a high accuracy and thus this method provides a powerful strategy in the fault diagnosis for the practical applications.



## 1. Introduction

Hydro-turbine is an important component in electric power generation. A fault in such a device could lead to huge economic losses and, therefore, it is desirable to conduct correct and efficient fault diagnosis on this facility. Fault diagnosis is performed when a hydro-turbine is malfunctioning. It can be used to determine the cause(s) responsible from a set of observed symptom(s). Vibration signals contain abundant information on the operation status of the mechanical equipment, and its analysis is of significance to the fault diagnosis of rotating machinery, including hydro-turbines. Generally, the analysis of vibration signals can be divided into two aspects: one is to further understand the fault mechanism, and the other is to extract effective features for fault diagnosis. Many techniques, like traditional FFT analysis, STFT analysis, and high order cumulate spectrum analysis, emphasize the frequency structure analysis of vibration signals. They aim at finding some efficient fault features from the vibration signals and have had good application in fault diagnosis. Combining these feature extraction methods with pattern recognition theories, such as neural network, GMM network, fuzzy logic network, Bayesian network, it is possible to realize intelligent on-line identification and fault diagnosis of machinery vibration signals [1–6]. However, these methods have shortcomings as they are more concerned about the frequency domain rather than the energy distribution and structural features. It is still a challenge to explore effective techniques that can extract specific features from vibration signals.

Vibration signals from hydropower stations usually demonstrate unstable and transient properties. Some are even random variation signals, and these signals show fractal features to some extent. The analysis of vibration signals has evolved with the development of the signal-processing method. Traditional fractal theory has been used in the fault diagnosis of hydropower units in recent years, and proved to be feasible [7,8]. It provides a geometric structure analysis method for complex signals, and has a number of successful applications in many fields. For example, chaotic fractal theory has been applied to the feature extraction of fault diagnosis of gas valves in Ref. [9] and a three-dimensional fractal measurement for a rock joint surface was conducted, and studied the relationship between

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surface fractal characteristic and hydro-mechanical behavior was studied in [10]. In fault diagnosis of mechanical equipment, fractal geometry method is used in vibration signal analysis with some fruitful outputs in [11].

The recently developed wavelet fractal algorithm has proven particularly applicable in the feature extraction of vibration signals [12]. However, its application is only found in the field of rotating machinery for its novelty. Considering the similarity between hydro-turbine vibration signals and rotating machinery vibration signals, it is felt that the wavelet fractal algorithm might be also applicable in the field of hydropower units. The principle of fractal-wavelet spectrum is simple: decompose vibration signals into different frequency components, then calculate the variance of each frequency band individually, which describes the energy distribution on each level, so as to describe the complexity and irregularity of signal on different scales and frequency bands. Wavelet fractal algorithm on several sets of simulated hydro-power vibration signals is conducted in this paper in two steps: first draw the power spectrum of the de-noised signal for the chosen wavelet function and decomposition level used in the algorithm; then decompose the signal and calculate the variance of selected decomposed levels, thus acquiring the fractal dimension after a few simple mathematical steps. For comparison, conventional fractal dimension calculating algorithm is also applied on the same set of data. The results show that although both methods are successful in feature extraction, the proposed wavelet fractal algorithm shows better performance in accuracy.

## 2. Fractal dimension and GP method

### 2.1. Fractal dimension

Non-linear dynamic and chaotic theory can be used to describe the irregular, broadband signals, which are generic in non-linear dynamic systems. They are effective in extracting some physically interesting and useful features from such signals. Fractal dimension, the capacity dimension, correlation dimension, and information dimension, developed from the non-linear dynamic and chaotic theory, is a promising new tool to interpret observations of physical systems with a fractal structure [12].

### 2.2. Definition of fractal dimension

There are quite a few classifications of fractal dimension: self-similarity dimension, Hausdorff dimension, box dimension, correlation dimension, information dimension, etc. The definition of fractal dimension differs as the classification changes [13]. In spite of the differences in the manner of their definitions, the essence is the same: measure the fractal diagram or signal on a certain scale, then express fractal dimension as the ratio of measuring result to the measuring scale. For instance, the self-similarity dimension $D$ is defined by fractal graphics with strict self-similarity as:

$$D = \lim_{\delta \to 0} \frac{\ln N}{\ln(1/r)} \tag{1}$$

Assume that the fractal entirety $S$ consists of $N$ non-overlapping parts $s_1, s_2, s_3, \ldots, s_N$, and each $s_i$ part is equal to the universal set $S$ after a $1/r_i$ $(0 < r_i < 1, i = 1, 2, \ldots, N)$ time amplification. The strict requirements above put a narrow limitation to the range of its application.

The fractal dimension of a set $S$ in a metric space, such as a geometric object or the phase space trajectory of a dynamic system, can be computed from several different measures. One of the most used measures is the correlation dimension. A widely used algorithm of correlation dimension is the G–P method proposed in Ref. [14]. Adaptive improvements have been made to the method, like the principle in the selection of reconstructed phase space's dimension, the value of the positive number $r$. There are principles to be followed when applying the method in a simulation or in practice, and they are introduced in Section 4.2.

### 2.3. Correlation dimension and Grassberger–Procaccia's algorithm

It is needed to estimate the dimension of an attractor $\mathcal{A}$ which is embedded in an m-dimensional Euclidean space from a sample of $N$ points on the attractor. That is, from the set $\{x_1, x_2, \ldots, x_n\}$ with $x_i \in \mathcal{A} \subset \mathbb{R}^m$. It is suggested in Ref. [13] to measure the distance between every pair of points and then compute the correlation integral:

$$C(N, r) = \frac{2}{N(N-1)} \sum_{\substack{i,j \\ (1 \le i < j \le N)}} H(r - \|x_i - x_j\|) \tag{2}$$

where $H(x)$ is the Heaviside step function. The summation counts the number of pairs $(x_i, x_j)$ for which the distance $\|x_i - x_j\|$ is less than a given positive number $r$.

The measure is obtained by considering correlations between points of a long-time series on the strange attractor. The practical calculation process of correlation dimension can be summarized as follows:

For a given one-dimension time series $\{x_1, x_2, \ldots, x_n\}$ with a fixed time increment $\tau$ and embedded dimension set to $m$, a matrix $X_{l\times m}$ is calculated by phase space reconstruction:

$$X_{l\times m} = [X_1, X_2, \ldots, X_l]^T \tag{3}$$

where $X_i = \{x_i, x_{i+1}, \ldots, x_{i+m-1}\}$, $i = 1, 2, \ldots, l$. $l = N + 1 - m$.

Take the row vector $X_i (i = 1, 2, \ldots, l)$ of $X_{l\times m}$ as the point of reconstructed phase space. Measure the spatial correlation degree with the correlation integral $C(r)$ defined according to:

$$C(r) = \frac{1}{l^2} \sum_{i=1}^{l} \sum_{j=1}^{l} \theta(r - \|X_i - X_j\|) \tag{4}$$

where $r$ is a fixed positive number, and $\theta(x)$ is the Heaviside function:

$$\theta(x) = \begin{cases} 1, & x \ge 0 \\ 0, & x < 0 \end{cases} \tag{5}$$