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Average intensity of four-petal Gaussian beams through paraxial optical system in atmosphere turbulence

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ABSTRACT

The propagation properties of four-petal Gaussian beams through the paraxial ABCD optical system in atmosphere turbulence are investigated by numerical examples. It is found that the beam propagating through ABCD optical system in atmosphere turbulence can keep its initial four-petal beam profile almost invariant for a short propagation distance, the beam will converted to Gauss-like beam with several small petals with the propagation increasing, and the beam loses its initial profile slower than the beam propagating in atmosphere turbulence.

1. Introduction

The propagation properties of various laser beams in atmosphere turbulence have been widely studied due to their essential applications in free-space/underwater optical communications and remote sensing [1]. In the past years, the average intensity and spreading properties of various laser beams, such as flat-topped beams, laser array beams, stochastic electromagnetic beams, partially coherent multiple Gaussian beams and modified Bessel–Gaussian beams etc., propagating through the turbulent atmosphere have been studied [2–6]. Recently, a new kind of beam called the four-petal Gaussian beam has been introduced to describe the laser beams with four petals at the source plane [7], and subsequently the vectorial nonparaxial properties, the beam propagation factor, far field properties etc. of four-petal Gaussian beams have been widely studied [8–12].

On the other hand, the optical system is also used to improve the optical properties when laser beams are applied to free-space/underwater optical communications and remote sensing. Therefore, the studies of the propagation properties for laser beams passing through an optical system in atmosphere turbulence are very important. The diffraction properties of a laser beam propagating through an optical system in atmosphere turbulence were first studied by Yura [13]. Since then, various laser beams propagating through the optical system in the turbulent atmosphere have been studied [14–18]. To the best of our knowledge, the four-petal Gaussian beam through an optical system in turbulent atmosphere has not been taken into account.

In this paper, we have derived the propagation formulae of four-petal Gaussian beam through the paraxial ABCD optical system in atmosphere turbulence. As an example, we study the average intensity of the four-petal Gaussian beam propagating through a double-lens system in atmosphere turbulence.

2. Theoretical analysis

The electric field of four-petal Gaussian beam propagating through the half free space $z \geq 0$, and the propagation beam propagates along the z-axis, the four-petal Gaussian beam at the source plane $z = 0$ can be expressed as [7]

$$E(x_0, y_0, 0) = \left(\frac{x_0 y_0}{w_0^2}\right)^{2n} \exp\left(-\frac{x_0^2 + y_0^2}{w_0^2}\right) \tag{1}$$

where n denotes the order of the four-petal Gaussian beam, w_0 denotes the waist width of Gaussian beam. If $n = 0$, Eq. (1) reduces to the well-known Gaussian beam with beam waist size w_0.

Based on the paraxial propagation theory, the propagation of electromagnetic beam through the ABCD optical system in

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atmosphere turbulence can be treated by using the extended Huygens–Fresnel diffraction integral [13]:

$$E(x,y,L) = \frac{1}{i\lambda B}\exp(ikL)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} E_0(x_0,y_0,0) \times \exp\left\{-\frac{ik}{2B}\left[A\left(x_0^2+y_0^2\right)+D\left(x^2+y^2\right)-2\left(xx_0+yy_0\right)\right]+\psi(x_0,y_0,x,y)\right\}dx_0dy_0 \quad (2)$$

where λ is the wavelength; A, B and D are the geometrical ray-matrix elements for the optical system; $\psi(x_0, y_0, x, y)$ is the solution to the Rytov method that represents the random part of the complex phase; $r = (x, y)$ and $r_0 = (x_0, y_0)$ are the position vectors at the output plane z and the input plane $z = 0$, respectively.

Then the average intensity of four-petal beams through an optical system in atmosphere turbulence can be expressed as follows:

$$\langle I(x,y,L)\rangle = \frac{1}{\lambda^2B^2}\iiiint_{-\infty}^{+\infty} E_x(x_{01},y_{01},0)E_x^*(x_{02},y_{02},0) \times \exp\left\{-\frac{ik}{2B}\left[A\left(x_{01}^2-x_{02}^2+y_{01}^2-y_{02}^2\right)-2\left(xx_{01}-xx_{02}+yy_{01}-yy_{02}\right)\right]\right\} \times \left\langle \exp\left[\psi(x_{01},y_{01},x,y)+\psi^*(x_{02},y_{02},x,y)\right]\right\rangle dx_{01}dy_{01}dx_{02}dy_{02} \quad (3)$$

where the asterisk $*$ denotes the complex conjugation, the $\langle\rangle$ indicates the ensemble average over the atmosphere turbulence statistics covering the log-amplitude and phase fluctuations due to the atmosphere turbulence. And

$$\left\langle \exp\left[\psi(x_{01},y_{01},x,y)+\psi^*(x_{02},y_{02},x,y)\right]\right\rangle = \exp\left[-\frac{(x_{01}-x_{02})^2+(y_{01}-y_{02})^2}{\rho_0^2}\right] \quad (4)$$

where ρ_0 is the spherical-wave lateral coherence radius due to the turbulence of the entire optical system and is defined as [13]

$$\rho_0 = B\left[1.46k^2C_n^2\int_0^L b^{5/3}(z)\,dz\right]^{-3/5} \quad (5)$$

where C_n^2 is the constant of refraction index structure of atmosphere turbulence and which describes the turbulence level of atmosphere turbulence. ρ_0 is the coherence length (induced by the atmosphere turbulence) of a spherical wave propagating in the atmosphere turbulence. $b(z)$ corresponds to the approximate matrix element for a ray propagating backwards through the system. L is the axial distance between the source plane and the output plane.

By substituting four-petal Gaussian beam Eq. (1) into Eq. (3), and recalling the following equation

$$\int_{-\infty}^{+\infty} x^n\exp\left(-px^2+2qx\right)dx = n!\exp\left(\frac{q^2}{p}\right)\left(\frac{q}{p}\right)^n\sqrt{\frac{\pi}{p}}\sum_{k=0}^{[n/2]}\frac{1}{k!(n-2k)!}\left(\frac{p}{4q^2}\right)^k \quad (6)$$

we can obtain the following expressions of four-petal Gaussian beams propagating through an optical system in atmosphere turbulence

$$I(x,y,z) = I(x,z)I(y,z) \quad (7)$$

where $I(x,z)$ and $I(y,z)$ are given by

$$I(x,z) = \frac{\pi(2n)!}{\lambda B(2w_0)^{4n}}\sum_{m=0}^{n}\sum_{t}^{2n-2m}(-1)^m a^{-2n+m-0.5}b^{-n-0.5t-0.5}\rho_0^{-2t} \left(-\frac{ikx}{B}\right)^{2n-2m-t}\frac{H_{2n+t}\left(-0.5ib^{-0.5}cx\right)}{m!t!(2n-2m-t)!}\exp\left(\frac{-2k^2w_0^2\rho_0^2x^2}{8w_0^2B^2+4B^2\rho_0^2+k^2A^2w_0^4\rho_0^2}\right) \quad (8a)$$

$$I(y,z) = \frac{\pi(2n)!}{\lambda B(2w_0)^{4n}}\sum_{m=0}^{n}\sum_{t}^{2n-2m}(-1)^m a^{-2n+m-0.5}b^{-n-0.5t-0.5}\rho_0^{-2t} \left(-\frac{iky}{B}\right)^{2n-2m-t}\frac{H_{2n+t}\left(-0.5ib^{-0.5}cy\right)}{m!t!(2n-2m-t)!}\exp\left(\frac{-2k^2w_0^2\rho_0^2y^2}{8w_0^2B^2+4B^2\rho_0^2+k^2A^2w_0^4\rho_0^2}\right) \quad (8b)$$

with

$$a = \frac{2B\rho_0^2 - ikAw_0^2\rho_0^2 + 2w_0^2B}{2w_0^2B\rho_0^2} \quad (9)$$

$$b = \frac{4B^2\rho_0^2 + 8w_0^2B^2 + k^2A^2w_0^4\rho_0^2}{4w_0^2B^2\rho_0^2 - 2ikAw_0^4B\rho_0^2 + 4w_0^4B^2} \quad (10)$$

$$c = \frac{2ikB\rho_0^2 + k^2Aw_0^2\rho_0^2}{2B^2\rho_0^2 - ikAw_0^2B\rho_0^2 + 2w_0^2\rho_0^2} \quad (11)$$

And $H_n(x)$ is the Hermite polynomial of order n, which has the following expression

$$H_n(x) = \sum_{k=0}^{[n/2]}\frac{(-1)^k n!}{k!(n-2k)!}(2x)^{n-2k} \quad (12)$$

By using the derived Eqs. (7)–(11), we can directly obtain the propagation properties of four-petal Gaussian beam propagating an optical system in atmosphere turbulence.

3. Numerical examples and analysis

In this section, we will study the propagation properties of four-petal Gaussian beam through the paraxial ABCD optical system in atmosphere turbulence. The calculation parameters are chosen as $w_0 = 5$ mm and $\lambda = 532$ nm. And as an example, the optical system of two thin lenses is considered (Fig. 1). The dimension of the lens are assumed to be larger than the beam diameter, then the ABCD parameters are $D = 1$, and

$$A = 1 - \frac{z_2+z_3}{f_1} + \frac{z_3(z_2-f_1)}{f_1f_2} \quad (13)$$

$$B = L - \frac{z_1(z_2+z_3)}{f_1} - \frac{z_1(z_1+z_2)}{f_2} + \frac{z_1z_2z_3}{f_1f_2} \quad (14)$$

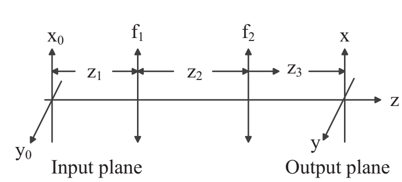

Fig. 1. Propagation schematic diagram of the paraxial optical system in atmosphere turbulence.