Optik 127 (2016) 3248–3252

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

# A study on the coupling coefficients for multi-core fibers

Wenhua Ren[a,b,*], Zhongwei Tan[a]

[a] Institute of Lightwave Technology, Key Laboratory of All Optical Network and Advanced Telecommunication Network of EMC, Beijing Jiaotong University, Beijing 100044, China

[b] Electrical Engineering Department, University of California Los Angeles, Los Angeles, CA 90095, USA

ARTICLE INFO

*Article history:*
Received 22 July 2015
Accepted 7 December 2015

*Keywords:*
Multi-core fibers
Coupled-mode theory
Coupling coefficients
Strong coupling

ABSTRACT

The complete analytical expressions of mode coupling dynamics for homogeneous 7-core multi-core fibers (MCFs) are obtained by using orthogonal coupled-mode theory (CMT). All the coupling coefficients between the adjacent cores, nonadjacent cores and self-coupling coefficients are taken into account, and proposed to be calculated by using their definition formulas rather than the frequently-used twin core fiber (TCF) model utilized in most publications before. Simulation results show that the coupling coefficients calculated by using the TCF model are smaller than that calculated by using definition formulas and consequently are not accurate, especially for strong coupling MCFs. Also the coupling coefficients between nonadjacent cores and self-coupling coefficients can be comparable to the coupling coefficients between adjacent cores, and consequently cannot be ignored in the analysis of strong coupling MCFs.



## 1. Introduction

There has been considerable interest in the study of multi-core fibers (MCFs) for their wide applications in optical communications [1–6], fiber lasers [7–10], fiber couplers [11,12], fiber sensors [13–15] and microwave photonics [16], etc. The mode coupling characteristic is one of the most basic properties of MCFs, which determines the characteristics of MCFs to some extent. In terms of optical communications, MCFs can be used in space-division multiplexing (SDM) [1–3] or mode-division multiplexing (MDM) [4–6] systems to overcome the limit of transmission capacity. For MCFs used in SDM systems usually there should be weak coupling among cores to avoid crosstalk [17,18], while strong coupling is necessary for the application in MDM systems to form super-modes [4–6]. The coupling characteristic is also the operational principle of fiber lasers and couplers based on MCFs. Consequently, a complete and intensive study of the mode coupling dynamics of MCFs is significant and necessary for the analysis, design and applications of MCFs.

In past years, the mode coupling dynamics for MCFs have been studied and discussed in numerous publications [5,11,17–19]. The coupled mode theory (CMT) can be used to analyze the mode coupling dynamics for MCFs directly and effectively [5,11,17–19], in which the coupling coefficients are key parameters. However, most of them only consider the mode coupling between adjacent cores [11,17], a reasonable assumption for weak coupling, but it will lead to error for strong coupling. In Ref. [18], all the coupling coefficients between adjacent cores, nonadjacent cores and self-coupling coefficients are taken into account in the analysis, but the discussion is still based on weak coupling MCFs, and the simulation results show that the coupling coefficients between nonadjacent cores and self-coupling coefficients can be ignored compared with that between adjacent cores. In Ref. [5], the CMT is used to analyze the mode coupling dynamics for strong coupling MCFs, but the self-coupling coefficients were not taken into account. What's more, all the earlier analyses utilized only the coupling coefficients obtained by using the twin core fiber (TCF) model [5,17,18,20], which is not accurate and even incorrect for MCFs.

In this paper, we analytically derive the complete mode coupling dynamics for homogeneous 7-core MCFs by using the coupled mode theory (CMT). All the coupling coefficients between adjacent cores, nonadjacent cores and self-coupling coefficients are taken into account and discussed. The rest of the paper is organized as follows. In Section 2, the general coupled-mode formalism for MCFs is obtained by using CMT. Furthermore, analytical expressions of mode coupling dynamics in homogeneous 7-core MCFs are given and discussed in Section 3. In Section 4, all the coupling coefficients are calculated and discussed using numerical simulation. Finally, conclusions are presented in Section 5.

It should be mentioned that in the following analysis, each core of the MCF is assumed to be single mode.

* Corresponding author at: Institute of Lightwave Technology, Key Laboratory of All Optical Network and Advanced Telecommunication Network of EMC, Beijing Jiaotong University, Beijing 100044, China. Tel.: +86 105 1684020.
*E-mail address:* whren@bjtu.edu.cn (W. Ren).

http://dx.doi.org/10.1016/j.ijleo.2015.12.021
0030-4026/

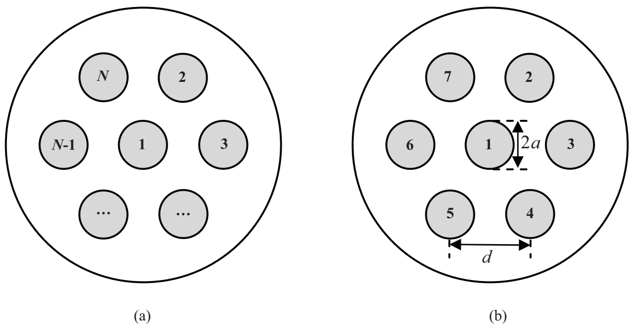


**Fig. 1.** (a) $N$-core MCF structure. (b) Homogeneous 7-core MCF structure.

## 2. General coupled-mode formalism for MCFs

Without loss of generality, first we consider an $N$-core MCF composed of a center core (labeled core 1) and $N-1$ surrounding cores (labeled core 2, ..., $N$), as in Fig. 1. If the mode field of each core is $E_k = A_k\exp(-i\beta_k z)$, where $A_k$ is the amplitude of the field at the $k$th core and $\beta_k$ is the propagation constant of the single core in the absence of other cores, the propagation of the fields in each core can be described by using the CMT

$$\frac{d\mathbf{E}}{dz} = -i\,\mathbf{C}\mathbf{E} \tag{1}$$

where $\mathbf{E} = [\,E_1 \quad \cdots \quad E_N\,]^{\boldsymbol{T}}$ and $\mathbf{C}$ is a $n \times n$ matrix with elements given by [17]

$$c_{mn} = \begin{cases} \kappa_{mn} & m \neq n, \\ \beta_m + M_m & m = n. \end{cases} \tag{2}$$

Here $\kappa_{mn}$ is the mode coupling coefficient between the core $m$ and $n$, and $M_m$ is the self-coupling coefficient of the mode field of core $m$, which are given by [20]

$$\kappa_{mn} = \frac{\omega\varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N_{\text{total}}^2 - N_n^2)A_m^* \cdot A_n dxdy}{4P_m},$$
$$M_m = \frac{\omega\varepsilon_0 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}(N_{\text{total}}^2 - N_m^2)A_m^* \cdot A_m dxdy}{4P_m}, \tag{3}$$

where $\omega$ is the free-space angular frequency of the light, $\varepsilon_0$ is the vacuum permittivity, $N_{\text{total}}$ is the refractive index distribution in the entire MCF, $N_n$ is the refractive index distribution of the $n$th fiber, $P_m$ is the optical power carried by the eigen mode in fiber $m$ before mode coupling, respectively.

The solution to Eq. (1) can be obtained as [17]

$$\mathbf{E}(z) = \sum_{n=1}^{N} c_n \mathbf{v}_n \exp(-\gamma_n z) \tag{4}$$

and

$$c_n = \mathbf{V}^{-1}\mathbf{E}(0), \tag{5}$$

where $\gamma_n$ is an eigenvalue of $\mathbf{C}$, $\mathbf{v}_n$ is the corresponding eigenvector and $\mathbf{V} = [\,\mathbf{v}_1 \quad \mathbf{v}_2 \quad \cdots \quad \mathbf{v}_N\,]$.

Eq. (4) is a general expression for the $N$-core MCF, which describes the mode coupling dynamics among individual cores as the light propagates. In fact, $\gamma_n$ is a propagation constant of a super-mode supported by the MCF structure, and $\mathbf{v}_n$ is the modal distribution vector of the corresponding super-mode.

For MCFs, normally the cladding index $n_{cl}$ is uniform, and core indices are a little higher than $n_{cl}$ and can be different from each other. It is worth noting that $(N_{\text{total}}^2 - N_n^2)$ in Eq. (3) is nonzero inside all the core regions except the $n$th core region, so the integration in Eq. (3) should be carried out in all the core regions except the $n$th core region, which is different from the TCF model utilized in most earlier publications, where the integration is only carried out in one core region area [5,17,18]. Though $A_m^* \cdot A_n$ is much smaller in other core regions than that in $m$th and $n$th core regions, and it is reasonable to carry out the integration only in the core $m$ region for weak coupling MCFs, it is insufficient for rigorous analysis, especially for strong coupling MCFs. Also the integration has straightforward physical meaning, i.e. the existence of other cores will affect the mode coupling between the two cores $m$ and $n$. In this paper, we will calculate the coupling coefficients by using the rigorous definition formulas as in Eq. (3) and compare the results with TCF model.

## 3. Mode coupling dynamics in homogeneous 7-core MCFs

To further investigate the mode coupling behaviors for MCFs, here we consider a common homogeneous 7-core MCF with hexagonal core distribution, as in Fig. 1(b). The core radius, core index, cladding index and propagation constant of every single fiber are $a$, $n_{co}$, $n_{cl}$ and $\beta$, respectively, so the coupling matrix $\mathbf{C}$ in Eq. (1) will be

$$\mathbf{C} = \begin{bmatrix} \beta+M_1 & \kappa_{12} & \kappa_{12} & \kappa_{12} & \kappa_{12} & \kappa_{12} & \kappa_{12} \\ \kappa_{12} & \beta+M_2 & \kappa_{23} & \kappa_{24} & \kappa_{25} & \kappa_{24} & \kappa_{23} \\ \kappa_{12} & \kappa_{23} & \beta+M_2 & \kappa_{23} & \kappa_{24} & \kappa_{25} & \kappa_{24} \\ \kappa_{12} & \kappa_{24} & \kappa_{23} & \beta+M_2 & \kappa_{23} & \kappa_{24} & \kappa_{25} \\ \kappa_{12} & \kappa_{25} & \kappa_{24} & \kappa_{23} & \beta+M_2 & \kappa_{23} & \kappa_{24} \\ \kappa_{12} & \kappa_{24} & \kappa_{25} & \kappa_{24} & \kappa_{23} & \beta+M_2 & \kappa_{23} \\ \kappa_{12} & \kappa_{23} & \kappa_{24} & \kappa_{25} & \kappa_{24} & \kappa_{23} & \beta+M_2 \end{bmatrix}. \tag{6}$$

The eigenvalues and corresponding eigenvectors of the matrix $\mathbf{C}$ can be shown to be [20]

$$\begin{aligned} &\gamma_1 = \beta + K + (M_1+M_2)/2 + C,\ \gamma_2 = \beta + K + (M_1+M_2)/2 - C, \\ &\gamma_3 = \gamma_7 = \beta + M_2 + \kappa_{23} - \kappa_{24} - \kappa_{25}, \\ &\gamma_4 = \gamma_6 = \beta + M_2 - \kappa_{23} - \kappa_{24} + \kappa_{25}, \\ &\gamma_5 = \beta + M_2 - 2\kappa_{23} + 2\kappa_{24} - \kappa_{25}, \end{aligned} \tag{7}$$

and

$$\begin{aligned} \mathbf{v}_1 &= \frac{1}{\sqrt{\left(\dfrac{K+\delta-C}{\kappa_{12}}\right)^2 + 6}}\left[\dfrac{-(K+\delta)+C}{\kappa_{12}} \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\right]^{\mathrm{T}}, \\ \mathbf{v}_2 &= \frac{1}{\sqrt{\left(\dfrac{K+\delta+C}{\kappa_{12}}\right)^2 + 6}}\left[\dfrac{-(K+\delta)-C}{\kappa_{12}} \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1\right]^{\mathrm{T}}, \\ \mathbf{v}_3 &= \frac{1}{2\sqrt{3}}\left[0 \quad 2 \quad 1 \quad -1 \quad -2 \quad -1 \quad 1\right]^{\mathrm{T}}, \\ \mathbf{v}_4 &= \frac{1}{2\sqrt{3}}\left[0 \quad 2 \quad -1 \quad -1 \quad 2 \quad -1 \quad -1\right]^{\mathrm{T}}, \\ \mathbf{v}_5 &= \frac{1}{\sqrt{6}}\left[0 \quad 1 \quad -1 \quad 1 \quad -1 \quad 1 \quad -1\right]^{\mathrm{T}}, \\ \mathbf{v}_6 &= \frac{1}{2}\left[0 \quad 0 \quad 1 \quad -1 \quad 0 \quad 1 \quad -1\right]^{\mathrm{T}}, \\ \mathbf{v}_7 &= \frac{1}{2}\left[0 \quad 0 \quad 1 \quad 1 \quad 0 \quad -1 \quad -1\right]^{\mathrm{T}}. \end{aligned} \tag{8}$$

where $\Delta M = M_2 - M_1 = 2\delta$, $K = \kappa_{23} + \kappa_{24} + \kappa_{25}/2$, $C = \sqrt{(K+\delta)^2 + 6\kappa_{12}^2}$. The parameter $K$ denotes the total coupling