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A no-equilibrium hyperchaotic system with a cubic nonlinear term

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ABSTRACT

Discovering dynamical systems with hidden attractors has become an attractive topic recently. A novel four-dimensional continuous-time autonomous system with a cubic nonlinear term is introduced in this work. It is worth noting that this no-equilibrium system can generate hidden hyperchaotic attractors. The fundamental properties of such systems are investigated by means of equilibrium points, phase portrait, bifurcation diagram and Lyapunov exponents. In addition, an adaptive scheme has been presented to synchronize two such identical hyperchaotic systems. Moreover, an electronic circuit is also designed and implemented to verify the feasibility of the theoretical model.

1. Introduction

Finding new chaotic systems is still an on-going topic because of potential chaos-based applications [1–3]. Numerous systems with chaotic behaviors have been reported in the literature [4–10]. Recently, chaotic system without equilibrium has attracted the interest of the scientific community because its special features. Wei introduced a simple three-dimensional (3D) autonomous system with no equilibrium based on Sprott D system [11]. A list of seventeen elementary 3D chaotic flows without equilibrium was reported in [12]. In addition, hyperchaotic systems with no equilibria were discovered [13,14]. It is noting that we cannot apply the Shilnikov method [15,16] to verify chaos in such no-equilibrium systems because they have neither homoclinic or heteroclinic orbits. Interestingly, such systems without equilibrium belong to the class of system with hidden attractors [17,18]. A hidden attractor cannot be investigated by applying an usual numerical approach in which a trajectory started from the neighborhood of an unstable equilibrium [19,20]. There has been increasing attention to systems with the presence of hidden attractors [21–27].

In the literature, an effective approach to construct new chaotic systems is using cubic nonlinearity. Chaotic behavior in cubic Chua's circuit system was presented in [28–30]. Zhong implemented Chua's circuit with a cubic nonlinearity where the basic circuit was a multiplier circuit with a feedback loop [31]. Cubic jerk systems and the simplest case were investigated [32–35]. Cubic nonlinearities were applied to create various chaotic systems [36–40]. In addition, chaotic attractor can be generated in time-delay systems containing a cubic polynomial [41–43]. Chen and Yang introduced a hyperchaotic system with a curve of equilibria by adding a feedback controller to the classical Lorenz system. This hyperchaotic system contained a cubic nonlinearity [44]. Moreover, hidden attractor has been observed in the no-equilibrium system with two cubic nonlinearities [45]. It raises the question of whether there are other systems with cubic nonlinear terms which can display hidden hyperchaotic attractors.

In this work, a novel no-equilibrium hyperchaotic system with a cubic nonlinearity is proposed. The paper is organized as follows. The theoretical model of the new no-equilibrium system is introduced in Section 2 while its properties are presented in Section 3. In Section 4, adaptive synchronization scheme for the identical novel no-equilibrium hyperchaotic systems is derived. Circuit implementation of such no-equilibrium system is studied detail in Section 5. Finally the last section presents the conclusion remarks.

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2. Model of the new no-equilibrium hyperchaotic system with a cubic nonlinearity

As have been known, Lorenz has introduced a famous model for atmospheric convection [46]. Lorenz model has the following simple three-dimensional continuous-time form

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = cx - y - xz, \\ \dot{z} = -bz + xy, \end{cases} \tag{1}$$

where x, y, z are state variables and a, b, c are parameters. Lorenz system can exhibit complex dynamics, especially chaotic behavior [46,2].

In order to generate hyperchaos from Lorenz system (1), its dimension has to be extended and two vital conditions [47] must be satisfied. Therefore, a new hyperchaotic system is generated by introducing a new term w as follows

$$\begin{cases} \dot{x} = a(y-x), \\ \dot{y} = cx - y - xz + w - d, \\ \dot{z} = -bz + xy, \\ \dot{w} = my + w - nx^3, \end{cases} \tag{2}$$

where x, y, z, w are state variables while a, b, c, d, m, n are positive parameters.

Interestingly, when $a=10$, $b=2$, $c=28$, $d=0.1$, $m=27$, $n=0.5$ and the initial condition $(x(0), y(0), z(0), w(0)) = (1, 0, 0, 0)$ are selected, new system (2) is hyperchaotic. In this case, the Lyapunov exponents of the system (2) are $\lambda_1 = 0.5602$, $\lambda_2 = 0.0445$, $\lambda_3 = 0$, and $\lambda_4 = -12.6016$. The projections of the hyperchaotic attractors are illustrated in Figs. 1 and 2.

In the next section, we indicate that there are no equilibrium points in the new system (2). Thus it can listed as a hyperchaotic system with hidden attractor according to a new classification of dynamical systems [20,18]. To the best of our knowledge few authors have found the hyperchaotic hidden attractors [45,26,14].

3. Properties of the new no-equilibrium system with a cubic nonlinearity

We consider new system (2) when $d \neq 0$. Its fundamental properties and dynamics, including equilibria, dissipativity, bifurcation diagram and Lyapunov exponents are reported.

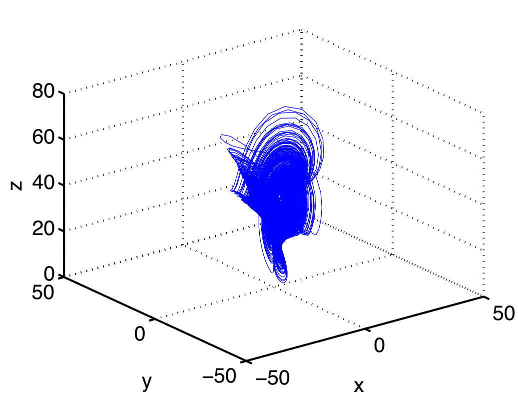

Fig. 1. The projection of the hyperchaotic attractor of the no-equilibrium system (2) in x–y–z space.

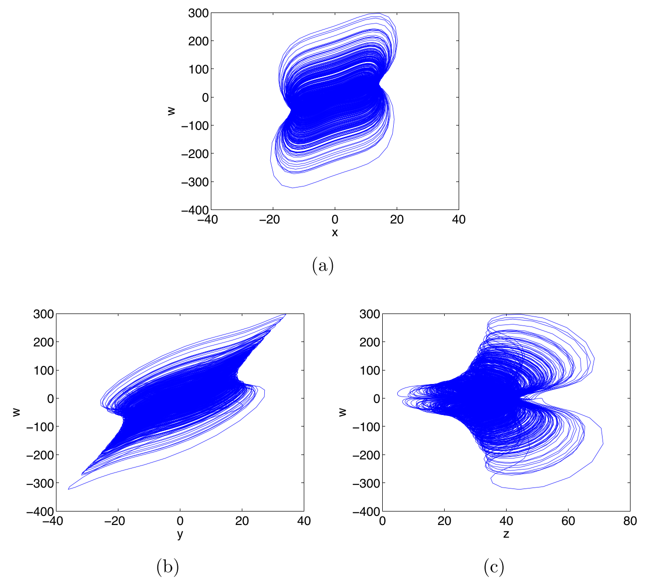

Fig. 2. The projection of the hyperchaotic attractor of the no-equilibrium system (2) in (a) x–w plane, (b) y–w plane, and (c) z–w plane.

It is easy to derive the equilibrium points for system (2) by solving $\dot{x} = 0$, $\dot{y} = 0$, $\dot{z} = 0$, and $\dot{w} = 0$,

$$a(y-x) = 0, \tag{3}$$

$$cx - y - xz + w - d = 0, \tag{4}$$

$$-bz + xy = 0, \tag{5}$$

$$my + w - nx^3 = 0. \tag{6}$$

For the set of selected parameters in Section 2, there are no equilibrium points for new system (2).

It is noting that new system (2) is dissipative with an exponential contractions rate

$$\frac{dV}{dt} = e^{-(-a-b)t}, \tag{7}$$

because

$$\nabla V = \frac{\partial \dot{x}}{\partial x} + \frac{\partial \dot{y}}{\partial y} + \frac{\partial \dot{z}}{\partial z} + \frac{\partial \dot{w}}{\partial w} = -a - b < 0. \tag{8}$$

Dynamics of the novel no-equilibrium system (2) with respect to the bifurcation parameter a are investigated. The bifurcation diagram in Fig. 3 is achieved by plotting the local maxima of the state

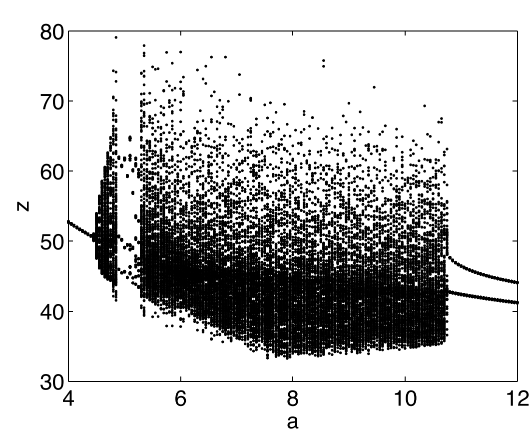

Fig. 3. Bifurcation diagram of z_{max} with $b=2$, $c=28$, $d=0.1$, $m=27$, $n=0.5$ and a as the varying parameter.