Optik 127 (2016) 5215–5218

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Entanglement dynamics of two qubits induced by a reservoir of coupled bosons

Zhongjie Wang*, Qijia Tai

Department of Physics, Anhui Normal University, Wuhu 241000, China

ARTICLE INFO

Article history:
Received 23 January 2016
Accepted 29 February 2016

Keywords:
Degenerate coupled bosons reservior
Entanglement dynamics
Concurrence

ABSTRACT

Entanglement dynamics of two qubits coupling to the degenerate coupled bosons reservoir (DCBR) have been analyzed. In the case of weak coupling, the envelope of the concurrence function shows exponential decay, and the stable concurrence shows oscillating change with coupling strength between modes. In the case of strong coupling, the concurrence function instead shows non-exponential decay in the initial short time.

1. Introduction

Entanglement of many-body quantum system has attracted much interest due to its fundamental importance in quantum information processing and quantum computation [1–3]. The uncontrolled interactions of the quantum system with its surrounding environment will result in irreversible decay of entanglement. In open quantum system theory, the environment is often represented as a collection of harmonic-oscillators (commonly referred to as a boson environment) [4]. Entanglement dynamics of qubits coupling to a boson environment have been investigated by using spin-boson model or a master equation of the Lindblad form [5–12]. It has been shown that in bipartite systems nonlocal decoherence (disentanglement) induced by environment, as opposite to the usual local decoherence, decays suddenly with time [13–18]. This phenomenon is called entanglement sudden death [ESD]. An experimental evidence about ESD has been reported recently by Almeida et al. [18]. ESD has been firstly analyzed [17] in a simple and realistic model where two initially entangled two-level atoms separately interact with the multimode vacuum noise of two distinct cavities. They found out that the nonlocal decoherence may take place suddenly or at least as fast as the sum of the normal single atom decay rates. This ESD has been analyzed also for two Jaynes–Cummings (JC) atoms [18], where the dynamics of the entanglement between the atomic internal variables shows different peculiarities for different initial states, and also shows sudden decaying of entanglement that are followed by periodic revivals.

In this paper, we investigate entanglement dynamics of two qubits interacting with a degenerate coupling bosons reservoir (DCBR). We analyze the feature of the time evolution of the concurrence in short time. For small coupling strength between modes, we find that under weak coupling conditions, the envelope of the concurrence function shows exponential decay, but under strong coupling conditions, the concurrence function shows. Gaussian decay.

2. DCBR model

Let us study the decoherence of two two-level atom (qubit) coupling to the environment formed by a set of N nearest-neighbor-interacting bosons with the same frequency ω [so called degenerate coupled bosons reservior (DCBR)]. We consider the qubit interacts all the bosons in the environment. The Hamiltonian of the total system is given by ($\hbar = 1$)

$$H = H_S + H_B + H_{SB} \tag{1}$$

$$H_S = \frac{\omega_a}{2}\sigma_z \tag{2}$$

$$H_B = \sum_{k=1}^{N} \omega b_k^+ b_k + J\sum_{k=1}^{N}\left(b_k^+ b_{k+1} + b_k b_{k+1}^+\right) \tag{3}$$

$$H_{SB} = \sum_{k=1}^{N}(\sigma_{z1} + \sigma_{z2})\left(g_k^* b_k + g_k b_k^+\right) \tag{4}$$

* Corresponding author. Tel.: +86 05533869013.
E-mail address: wuliwzj@mail.ahnu.edu.cn (Z. Wang).

http://dx.doi.org/10.1016/j.ijleo.2016.02.082
0030-4026/

where ω_a is the atomic transition frequency, σ_{zj} $(j=1,2)$ is the Pauli operator of the jth qubit, b_k and b_k^+ are annihilation and creation operators for the kth mode of the bath, respectively. J is intramode coupling parameter, g_k is coupling coefficient of the atom to the kth mode of the bath. It is noted that the environment Hamiltonian H_B [to see Eq. (3)] contains two terms: the first term is the free Hamiltonian, the second term describes the interacting Hamiltonian between bath modes. In the following, we diagonalize the Hamitonian H_B. Making Fourier transformation for the operators b_k and b_k^+

$$b_k = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\exp\left\{i\frac{2\pi jk}{N}a_j\right\} \tag{5}$$

$$b_k^+ = \frac{1}{\sqrt{N}}\sum_{j=1}^{N}\exp\left\{-i\frac{2\pi jk}{N}a_j^+\right\} \tag{6}$$

we can obtain the transformed the environment Hamitonian H_B as

$$H_B = \sum_{k=1}^{N}\Omega_k a_k^+ a_k \tag{7}$$

where $\Omega_k = \omega + 2J\cos\left(2k\pi/N\right)$, a_j^+ and a_j are boson operators. The interaction Hamiltonian H_{SB} described by a_j^+ and a_j can be represented as

$$H_{SB} = \sum_{k=1}^{N}(\sigma_{z1}+\sigma_{z2})\left(G_k^* a_k + G_k a_k^+\right) \tag{8}$$

where $G_k = \sum_{j=1}^{N} g_k e^{-i\pi jk/N}$. It is noted that G_k and g_k constitute a pair of the Fourier transformation relationship. In the interaction picture, the interaction Hamiltonian is

$$H_{SB}(t) = \sum_{k=1}^{N}(\sigma_{z1}+\sigma_{z2})\left(G_k^* e^{-it\Omega_k}a_k + G_k e^{it\Omega_k}a_k^+\right) \tag{9}$$

The evolution operator of the system is obtained by the following equation:

$$\begin{aligned} U(t,0) &= Te^{-i\int_0^t H_{SB}(s)ds} \\ &= \exp\left\{\frac{(\sigma_{z1}+\sigma_{z2})}{2}\sum_{k=1}^{N}\left[\xi_k(t)a_k^+ - \xi_k^*(t)a_k\right]\right\} \end{aligned} \tag{10}$$

where T is timing order operator, $\xi_k(t) = 2G_k(1-e^{it\Omega_k})/\Omega_k$. In order to derive the density operator of the qubit at any time, we first assume that the initial density operator of the system is $\rho(0) = \rho_S(0)\otimes\rho_E(0)$, $\rho_S(0)$ and $\rho_E(0)$ are the initial density operator of the qubit and the reservior, respectively. $\rho_E(0)$ May be represented as

$$\rho_E(0) = \prod_{k=1}^{N}\frac{e^{-\beta b_k^+ b_k}}{1-e^{\beta\omega}} = \prod_{k=1}^{N}\rho_{E,k}(\beta) \tag{11}$$

where $\rho_{E,k}\left(\beta\right) = e^{-\beta a_k^+ a_k}/\left(1-e^{-\beta\omega}\right)$, $\beta = 1/k_B T$ with k_B being Boltzmann constant, T being absolute temperature. The density operator of the qubit at any time is

$$\rho_S(t) = Tr_E\left\{U(t,0)\rho_S(0)\otimes\rho_E(0)U^+(t,0)\right\} \tag{12}$$

where $Tr_E\{\ldots\}$ denotes trace of the environment variables. In order to study the dynamics of entanglement in our model, we employ the concurrence as a entanglement measure. The concurrence is defined as [15]

$$C(\rho_S) = \max\left\{0, \lambda_1 - \lambda_2 - \lambda_3 - \lambda_4\right\} \tag{13}$$

where $\lambda_i(\lambda_1 \geq \lambda_2 \geq \lambda_3 \geq \lambda_4)$ are the eigenvalues of the time-dependent operator $\rho_S\left(\sigma_y\otimes\sigma_y\right)\rho_S{}^*\left(\sigma_y\otimes\sigma_y\right)$. The concurrence ensures the scale between 0 and 1. In particular, $C(\rho_S) = 1$ indicates maximum entanglement between the two qubits, whereas $C(\rho_S) = 0$ represents disentanglement. In order to compute the concurrence. In the subspace spanned by the basic atomic vectors

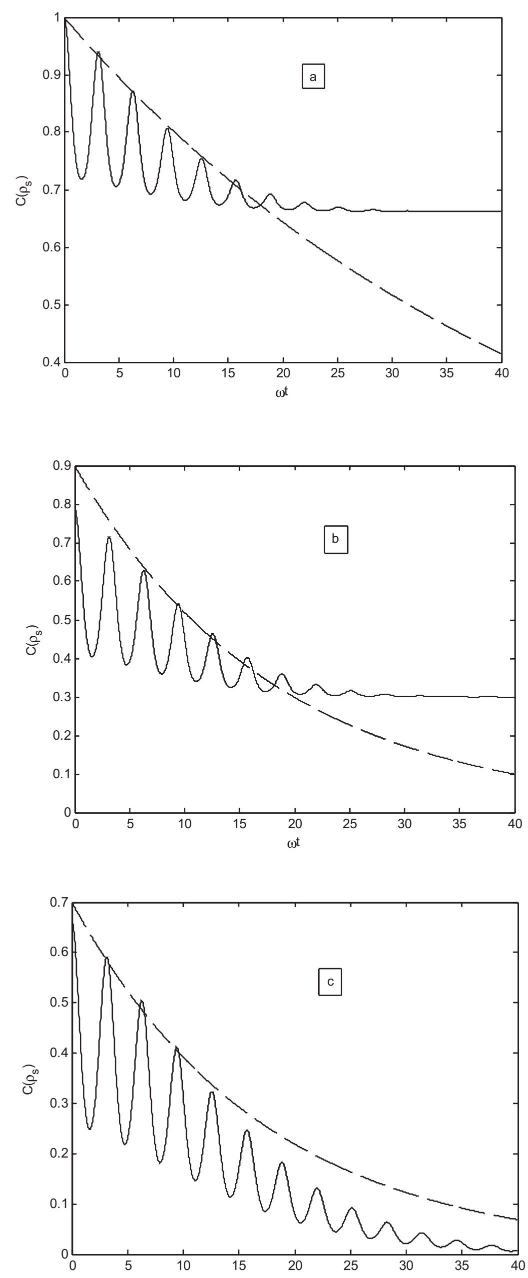

Fig. 1. The evolution of the concurrence with the scaled time $\tau = \omega t$ for the different parameter m with $g/\omega = 0.01$, $J/\omega = 0.5$, (a) $m = 0$, the dotted line corresponds to the exponential function $\exp(-0.022\omega t)$, (b) $m = 0.5$, the dotted line corresponds to the exponential function $0.9\exp(-0.055\omega t)$, (c) $m = 1$, the dotted line corresponds to the exponential function $0.7\exp(-0.058\omega t)$.