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Absorptive central line-shapes of a driven atom in a squeezed reservoir

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ABSTRACT

Amplification of absorption spectrum of a weak signal probing the system of a dissipative single 2-level atom in the presence of a broadband squeezed vacuum (SV) reservoir and driven by a non-resonant mono- or bi-chromatic coherent field is investigated. The non-autonomous model Bloch equations yields solutions for the atomic variables of all harmonic frequency components, $\exp(\pm in\nu t)$; $n = 0, 1, \ldots$ where ν is the probe frequency detuning. Amplification of the fundamental ($n = 0$) and the first harmonic ($n = 1$) frequency components associated with the central part ($|\nu| < 1$) of the spectrum is shown with: (i) simultaneous change of atomic and probe frequency detunings, and (ii) change of SV phase and photon number parameters.

1. Introduction

Absorption spectrum of a weak field probing a strongly driven dissipative 2-level atom, as theoretically predicted [1] and experimentally verified at optical frequencies [2], shows that the absorption line-shape is amplified (i.e. negative absorption) at the expense of the strong driving (pump) field in the vicinity of the Rabi frequency side bands, together with a central-dispersive line shape. The amplification of the side bands is due to dressed state population inversion [3], while the amplification of the central dispersive-like structure is due to coherent population oscillation as a result of the superposition of pump and probe fields beating at their frequency mismatch [4–6].

The damping of a 2-level atom driven by mono- or bi-chromatic pump field in the presence of a non-resonant squeezed vacuum (SV) reservoir has been recently analysed [7–9]. The corresponding non-autonomous model Bloch equations have been treated iteratively with arbitrary pump field strength. In particular, the resonance fluorescence spectrum is studied in the mono- [7] and bi-chromatic [8] cases. The absorption-dispersion spectra, in both cases of mono- and bi-chromatic pump fields, was analysed in great detail [9] with the purpose of identifying the regions of zero absorption (isolines) accompanied with finite or steep varied dispersion within the medium. This is related to the refractive index of the medium and hence to the propagating speed of the probe (pulse) field (see references in [9] concerning earlier studies in the normal vacuum case).

The main purpose of the present work is to investigate the possibility of amplifying the central component of the absorption line-shapes for a strongly driven 2-level atom by: (i) monochromatic field and, (ii) bichromatic field in the presence of an SV reservoir. The SV reservoir is considered non-resonant in case (i), while in case (ii) it is considered both resonant and non-resonant. Investigation of the central component of the absorption spectrum in the monochromatic driving case was given in [10] for exact resonant SV reservoir. The paper is presented as follows. The non-autonomous model of Bloch equations in both cases of mono- and bi-chromatic driving cases are introduced in Sections 2 and 3, respectively, together with their Fourier decomposition treatments and computational investigation of the central absorption line-shapes. A summary is given in Section 4.

2. Monochromatic driven case

The c-number non-autonomous model of Bloch equations for a single two-level atom driven by a strong monochromatic laser field in the presence of an off-resonant SV field and a weak probe field are of the form [9]:

$$\dot{r_+} = -(\Gamma + i\Delta)r_+ - |M|e^{i\phi}e^{iqt}r_- + (\Omega + \Omega_p e^{i\nu t})r_z \tag{1a}$$

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$$\dot{r_-} = \dot{r_+}^* \tag{1b}$$

$$\dot{r_z} = -\frac{1}{2} - 2\Gamma r_z - \frac{1}{2}(\Omega + \Omega_p e^{-i\nu t})r_+ - \frac{1}{2}(\Omega + \Omega_p e^{i\nu t})r_-, \tag{1c}$$

where $\dot{r}_{\pm,z} = \frac{d}{dt}r_{\pm,z}$.

The variables $r_{\pm,z}(t)$ are the mean atomic polarisation components and inversion, respectively. The rest of the symbols in (1) are: Ω is the (real) Rabi frequency associated with the strong field, Ω_p is the real Rabi frequency of the probe field, $\Delta = \omega_l - \omega_o$ is the atomic detuning with ω_l and ω_o are the frequencies of the laser field and atom respectively, $\nu = \omega_p - \omega_l$ is the beating mismatch frequency between the probe field (ω_p) and the laser field (ω_l), $q = 2(\omega_c - \omega_l)$ is the SV detuning parameter with ω_c is the central (carrier) frequency of the SV field, $\Gamma = \gamma(1+2N)/2$ and γ is the A-coefficient. $M = |M|e^{1\phi}$ and N are the SV field parameters: N is the average photon number and the (complex) M is a measure of degree of squeezing and ϕ is the relative phase of the SV field with respect to the laser field; $|M| \le \sqrt{N(N+1)}$. For a resonant SV field ($q = 0$), Eqs. (1) have been analysed in [10]. In general, Eqs. (1) with non-equal detuning parameters ($q \ne \nu$) are too lengthy to handle with Fourier decomposition approach. Here, we specify the case of non-resonant SV where $q = \nu \ne 0$ (hence, the results for resonant ($q = 0$) SV [10] cannot be obtained as a special case from our results).

To analyse the steady state central components of the absorption line-shape of the probe field (i.e. $\frac{\Omega_p}{\gamma}, \frac{\nu}{\gamma} \ll 1$), we resort to Fourier decomposition of (1) with $q = \nu \ne 0$ in the steady state where $\dot{r}_{\pm,z} = 0$, up to the first harmonic in $e^{\pm i\nu t}$. Further, as long as all higher harmonics $e^{\pm 3i\nu t}, \ldots$ are discarded we adopt a less laboured approach of Fourier decomposition (c.f. [11,12] also [10]) as follows. First, we set $\dot{r}_\pm = 0$ in (1a) and (1b) and then solve for $r_\pm$ and in turn substitute the results into (1c) to get the following:

$$r_- = [(\Gamma' + i\Delta')(\Omega' + \Omega'_p e^{-i\nu'\tau}) - |M|e^{-i\phi}(\Omega'_p + \Omega' e^{-i\nu'\tau})]\frac{r_z}{H} \tag{2a}$$

$$= (r_+)^* \tag{2b}$$

$$\frac{dr_z}{d\tau} = -\frac{1}{2} - (u_o + u_c \cos\nu'\tau + u_s \sin\nu'\tau)r_z, \tag{2c}$$

where:

$$\begin{aligned} u_o &= 2\Gamma' + \frac{\Omega'}{H}(\Omega'\Gamma' - 2\Omega'_p|M|\cos\phi) \\ u_c &= \frac{\Omega'}{H}(2\Gamma'\Omega'_p - \Omega'|M|\cos\phi) \\ u_s &= \frac{|M|\Omega'^2 \sin\phi}{H} \\ H &= \Delta'^2 + \frac{1}{4}, \end{aligned} \tag{3}$$

with the normalised quantities: $\tau = \gamma t$, $\Gamma' = \frac{\Gamma}{\gamma}$, $\Omega' = \frac{\Omega}{\gamma}$, $\Omega'_p = \frac{\Omega_p}{\gamma}$ and $\Delta' = \frac{\Delta}{\gamma}$.

In view of the harmonic coefficients in (2c), its solution in the steady state can assume the following Fourier ansatz (up to first harmonic $e^{\pm i\nu'\tau}$):

$$r_z = a_o + a_{\nu'}e^{i\nu'\tau} + a_{-\nu'}e^{-i\nu'\tau}, \tag{4}$$

where the coefficients $a_{o,\pm\nu'}$ are time-independent. By substituting Eq. (4) into Eq. (2c), and comparing coefficients of equal harmonics we get:

$$a_o = -\frac{(u_o^2 + \nu'^2)}{2u_o H_1} \tag{5a}$$

$$a_{\nu'} = \frac{u_c(u_o - i\nu') - u_s(\nu' + iu_o)}{4u_o H_1} = (a_{-\nu'})^* \tag{5b}$$

where:

$$H_1 = (u_o^2 + \nu'^2) - \frac{u_c^2 + u_s^2}{2}. \tag{5c}$$

The effect of the steady state population oscillations (terms in $a_{\pm\nu'}$) on the absorption line-shape can be seen by substituting Eq. (4) into Eq. (2a) and obtain (up to 1st harmonics $e^{\pm i\nu'\tau}$):

$$r_- = b_0 + b_{-\nu'}e^{-i\nu'\tau} + b_{\nu'}e^{i\nu'\tau}, \tag{6}$$

where the coefficient (amplitude) of the harmonic term $e^{-i\nu'\tau}$, is given by:

$$b_{-\nu'} = \frac{Aa_{-\nu'} + Ba_o}{H}, \tag{7a}$$

while the non-harmonic term is given by:

$$b_o = \frac{Aa_o + Ba_{\nu'}}{H}, \tag{7b}$$

where:

$$\begin{aligned} A &= (\Gamma'\Omega' - |M|\Omega'_p \cos\phi) + i(\Delta'\Omega' + |M|\Omega'_p \sin\phi), \\ B &= (\Gamma'\Omega'_p - |M|\Omega' \cos\phi) + i(\Delta'\Omega'_p + |M|\Omega' \sin\phi). \end{aligned} \tag{8}$$

Amplification of the central (non-harmonic and first harmonic) component amplitudes of the absorption line-shape, $Im(b_o)$ and $Im(b_{-\nu'})$, respectively, are shown for fixed strong and probe fields ($\Omega' = 10, \Omega'_p = 0.5$) and various system parameters in Fig. 1. Here, in all cases the amplification of the non-harmonic component is larger than that of the first harmonic component, as follows:

(a) The variation against the (small central) probe detuning $\nu' \in (-1,\ 1)$ with rest of parameters kept fixed shows that both components exhibit almost constant amplification (Fig. 1a);
(b) For simultaneous variation of probe and atomic detuning ($\nu' = \lambda\Delta'$; $|\lambda| = 10^{-3}$; $|\nu'| < 1$) both components exhibit dispersive-like structure and the amplification is best shown with SV parameters, $N = 0.1$, $\phi = 0$ and for $\Delta' > 0$ (Fig. 1b). For $\phi = \pi$, the amplification is shown for $\Delta' < 0$;
(c) The variation of the SV phase ϕ (Fig. 1c) shows that the non-harmonic component is constantly amplified with $\Delta' = 10$ over the whole interval, $0 \le \phi \le 2\pi$, while the first harmonic component is amplified over some interval of ϕ that depends on the value of Δ';
(d) Varying the SV average photon number N (Fig. 1d) shows that the non-harmonic component with $\phi = 0$ and $\Delta' = 10$ is best amplified with weak $N \simeq 0.1$, while the first harmonic component is best amplified with $\phi = \frac{\pi}{2}$ and $\Delta' = 0$.

3. Bichromatic driven case

In the case where the driving strong field is a bichromatic laser field of two frequency component $\omega_1, \omega_2 (\omega_1 \ne \omega_2)$, and of equal constant amplitudes Ω, the corresponding non-autonomous Bloch equations for the mean atomic variables have the form [9]:

$$\dot{r_+} = -(\Gamma + i\Delta)r_+ - |M|e^{2ik\delta t}e^{i\phi}r_- - i(2\Omega\cos\delta t + \Omega_p e^{i\nu t})r_z \tag{9a}$$

$$\dot{r_-} = \dot{r_+}^* \tag{9b}$$

$$\dot{r_z} = -\frac{1}{2} - 2\Gamma r_z - i(\Omega\cos(\delta t) + \Omega_p e^{-i\nu t})r_+ + i(\Omega\cos(\delta t) + \Omega_p e^{i\nu t})r_-. \tag{9c}$$