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Mean-square practical stability for uncertain stochastic system with additive noise controlled by optimal feedback

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ABSTRACT

Stability concepts addressed in the framework of Lyapunov are not suitable to analyze the stability of stochastic systems with additive noise since it has no equilibrium. The mean-square practical stability is introduced to study the stability of uncertain stochastic systems with additive noise controlled by linear-quadratic optimal feedback. By using Lyapunov functional methods and the comparison principle, criteria on mean-square practical stability of stochastic systems with partially known uncertainties and norm-bounded parameter uncertainties are deduced, respectively. Some numerical examples and simulations are given to illustrate the validity of the theoretical analysis.

1. Introduction

Linear-quadratic (LQ) optimal control theory for stochastic systems, pioneered by Wonham [1], is very significant and can be widely applied to the real word. Among the different issues related to stochastic LQ optimal control, stability analysis is an important research direction [2,3].

On the ground of Lyapunov direct method, several sophisticated stability concepts of Itô-type stochastic differential equations were investigated theoretically [3]. Meanwhile, considerable efforts has being devoted continuously to extending the theoretical stability concepts to the practical problems [4]. However, the stability concepts talked above are addressed in the framework of Lyapunov, which are established based on a standard assumption that the equilibrium of the system is exist. This is not practical because many systems in real life are driven by additive noise, which means that they do not have any equilibrium [5]. Ref. [6] analyzed the stability of these systems via taking mathematical expectation of the system thus eliminating the noise. This method actually ignore the noise term, which makes the results cannot be widely applied in practice.

Practical stability, proposed by LaSalle and Lefschetz [7], is motivated by the fact that the state of a physical system may be mathematically unstable, but it operate sufficiently near the desired state. This concept can be employed to analyze the stability of systems without equilibrium. Many literature have used practical stability to solve various problems [8–10]. In a series of papers, Xu and Zhai et al. studied practical stability for switched systems whose subsystems have no common equilibrium [11–13]. As for uncertain stochastic systems, to the best of our knowledge, few results were reported.

In this paper, mean-square practical stability (MSPS) is introduced to investigate uncertain Itô-type stochastic systems with additive noise under LQ optimal feedback for the nominal system. The criteria for two kinds of uncertain stochastic systems are derived and the main challenge is to deduce and prove the criteria.

The rest of this paper is organized as follows: Section 2 we introduce the formulation of the stochastic LQ optimal control problem and give some definitions. The MSPS criteria for two types of uncertain stochastic systems with LQ optimal feedback are deduced in Section 3. The paper ends with the discussion of numerical examples (Section 4) and some concluding remarks (Section 5). For convenience, throughout the paper, we adopt the following notations.

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Notation

- $\mathbb{R}^{m\times n}$: A $m \times n$ dimensional real Euclidean space.
- $\mathbb{R}_+$: The set of all nonnegative real numbers.
- $C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R})$: The set of all real-valued functions $V(x, t)$ defined on $\mathbb{R}^n \times \mathbb{R}_+$ and they are continuously twice differentiable in $x \in \mathbb{R}^n$ and once in $t \in \mathbb{R}_+$.
- A^T: The transpose of matrix A.
- $tr(A)$: The trace of matrix A.
- Let $\omega(t)$ be a 1-dimensional standard Brownian motion defined on the complete probability space $(\Omega, \mathcal{F}, P)$, with Ω being a sample space, $\mathcal{F}$ being a σ-field and P being a probability measure. Consider the following stochastic system:

$$dx(t) = f(x, t)\,dt + g(x, t)\,d\omega(t) \tag{1}$$

For any given $V(x, t) \in C^{2,1}(\mathbb{R}^n \times \mathbb{R}_+, \mathbb{R})$, a differential operator $\mathcal{L}$ associated with system (1) is defined as

$$\mathcal{L}V(x, t) = \frac{\partial V}{\partial t} + \left(\frac{\partial V}{\partial x}\right)^T f(t, x) + \frac{1}{2} tr\left(g^T \frac{\partial^2 V}{\partial x^2} g\right)$$

2. Problem formulation and preliminaries

Consider the following uncertain Itô-type stochastic system with additive noise

$$\begin{cases} dx(t) = [(A + \Delta A)x(t) + (B + \Delta B)u(t)]dt + (H + \Delta H)d\omega(t) \\ x(t_0) = x_0 \end{cases} \tag{2}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $H \in \mathbb{R}^n$ are constant matrices; $\Delta A \in \mathbb{R}^{n\times n}$, $\Delta B \in \mathbb{R}^{n\times m}$ are time-varying uncertainties; $x(t) \in \mathbb{R}^n$ is system state; $u(t) \in \mathbb{R}^m$ is control input.

Associated with the nominal system of uncertain system (2), a quadratic cost functional is given as

$$J = E\left[\int_0^\infty (x^T(t)Qx(t) + u^T(t)Ru(t))dt\right] \tag{3}$$

where the state weighting matrix Q is nonnegative-definite and the control weighting matrix R is positive definite.

According to the stochastic optimal control theory, an optimal feedback control of the above LQ problem under the condition of complete information is

$$u(t) = -R^{-1}B^T P(t)x(t) \tag{4}$$

here $P(t)$ is a nonnegative-definite matrix, which can be solved elegantly via the classical algebraic Riccati equation (ARE)

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{5}$$

Before stating the underlying problem, the following two definitions are given with reference to Ref. [7].

Definition 1 *((MSPS))*. The uncertain stochastic system (2) under optimal feedback (4) is called mean-square practically stable if, given real number pair (λ, Λ) with $\Lambda > \lambda > 0$, $E||x(t_0)||^2 < \lambda$ implies $E||x(t)||^2 < \Lambda$ for some $t_0 \in \mathbb{R}_+$. Here, $x(t) = x(t, t_0, x_0)$ is the solution process of system (2).

Definition 2 *((MSUPS))*. The uncertain stochastic system (2) under optimal feedback (4) is called mean-square uniformly practically stable if MSPS holds for every $t_0 \in \mathbb{R}_+$.

The following lemma and theorem will be used later to deduce the criterion.

Lemma 1 ([3]). Assume that $x \in \mathbb{R}^p$, $y \in \mathbb{R}^q$ and ε is an arbitrary positive number, the following inequality holds:

$$x^T y + xy^T \le \varepsilon x^T x + \frac{1}{\varepsilon} y^T y \tag{6}$$

Theorem 1 ((Rayleigh–Ritz Theorem) [14]). Let A be a n-dimensional Hermitian matrix. Then its eigenvalues are critical points of the Rayleigh-Ritz quotient, which is a real function as

$$R(x) = \frac{x^* Ax}{x^* x}$$

where x is a n-dimensional vector satisfying $||x|| \neq 0$ and x^* is conjugate transpose of x.

Let $\lambda_1 \ge \lambda_2 \ge \cdots \ge \lambda_n$ be the eigenvalues of matrix A, then the following inequalities hold

$$\lambda_n x^T x \le x^T Ax \le \lambda_1 x^T x \tag{7}$$

3. Robust mean-square practical stability

In this section, by employing the Lyapunov functional method and the comparison principle, the criteria on MSUPS of closed-loop LQ optimal control stochastic systems with partially known uncertainties and norm-bounded parameter uncertainties are deduced, respectively. The system is described as Eq. (2).