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Original research article

Fuzzy modeling, stabilization and synchronization of multi-scroll chaotic systems

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ABSTRACT

A novel fuzzy control concept for stabilizing and synchronizing multi-scroll chaotic systems is presented in this paper. First, the multi-scroll chaotic systems are exactly derived using Takagi-Sugeno (T-S) fuzzy models. Then, the fuzzy controllers for stabilization and synchronization processes are designed via the prediction-based approach. Computer simulations on typical multi-scroll chaotic systems are provided to show the effectiveness of the proposed schemes.

1. Introduction

Chaos is considered as the seemingly behavior of a deterministic system which shows an extreme sensitivity to initial conditions [1]. Recent research results include modifying the nonlinear characteristics of chaotic systems to generate the so-called multi-scroll attractors [2–8]. This interest was essentially caused by the fact that they are particularly suitable for practical applications such as random bit generation and secure communication [9–12]. On the other hand, much attention has been focused on stabilizing and synchronizing chaotic systems since the revolutionary works of Ott et al. [13], Pecora and Carroll [14]. Up to now, different methods and techniques have been proposed to stabilize and synchronize chaotic systems such as delayed feedback method [15], impulsive method [16–18], high order method [19,20], predictive method [21–23], back stepping method [24,25], adaptive method [26–28], and so on. However, stabilization and synchronization of multi-scroll chaotic systems have not been investigated enough. The control of such systems was first considered in [29] and then investigated in more details in [30–32]. We note that the method proposed in [31,32] allows stabilization of the typical multi-scroll Chua and Chen systems on their unstable fixed points and unstable periodic orbits with very satisfying performance.

In recent years, fuzzy control and synchronization of chaotic systems is attracting more attention [33–40]. This is due to the fact that the model of chaotic systems is generally uncertain and/or unknown in real-word applications. Takagi-Sugeno (T-S) type of fuzzy models was attested to be a very useful technique for modeling complex nonlinear systems such as chaotic systems.

The main contribution of this paper is to develop a rigorous and analytical method for the stabilization and synchronization of multi-scroll chaotic systems by a simple combination of predictive scheme and T-S fuzzy model. To our best knowledge, there has never been a study on stabilization and synchronization of multi-scroll chaotic systems using T-S fuzzy model. First, the multi-scroll chaotic system is exactly derived using a T-S fuzzy model. Then, based on the obtained T-S fuzzy model, a fuzzy predictive controller is proposed, which guarantees the global asymptotic stability of the unstable equilibrium points. Furthermore, the global asymptotic synchronization is also investigated. Simulation results are provided to demonstrate the effectiveness of the proposed method.

The structure of this paper is as follows. In Section 2, we introduce the T-S fuzzy model of two typical multi-scroll chaotic systems: the n-scroll Chua's circuit and the multi-scroll Chen system. Stabilization of equilibrium points of the two multi-scroll chaotic systems using the proposed fuzzy predictive controller will be discussed in Section 3. Synchronization process will be investigated in Section 4. Finally, concluding remarks are given in Section 5.

2. Fuzzy modeling of multi-scroll chaotic systems

Consider the following system

$$\dot{x}(t) = f(x(t)), \tag{1}$$

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where $x \in \mathbb{R}^n$ is the state variable, $f(x(t)) : \mathbb{R}^n \to \mathbb{R}^n$ is a nonlinear function with appropriate dimension. The T-S fuzzy model of (1) is described by fuzzy IF-THEN rules as follows:

$$\begin{aligned} &R^i: \ \text{IF} \ z_1(t) \ \text{is} \ M_{i1} \ \text{and} \cdots \text{and} \ z_p(t) \ \text{is} \ M_{ip} \\ &\quad \text{THEN} \ \dot{x}(t) = A_i x(t), \quad i = 1, \ldots, r, \end{aligned} \tag{2}$$

where R^i $(i = 1, \ldots, r)$ denotes the ith fuzzy rule, r is the number of fuzzy rules, $z_1(t), \ldots, z_p(t)$ are proper state variables, $M_{ij}(j = 1, \ldots, p)$ are fuzzy sets and A_i are system matrices with appropriate dimensions. The final output of the fuzzy system in inferred as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(z(t)) \left\{ A_i x(t) \right\}, \tag{3}$$

where $h_i(z(t)) = \prod_{j=1}^{p} M_{ij}(z_j(t))$ such that $\sum_{l=1}^{r} h_i(z(t)) = 1$ and $h_i(z(t)) \geq 0$.

2.1. Fuzzy modeling of the n-scroll Chua's circuit

For the n-scroll Chua's circuit [3]

$$\begin{cases} \dot{x}_1 = \alpha(x_2 - g(x_1)), \\ \dot{x}_2 = x_1 - x_2 + x_3, \qquad \text{with} \\ \dot{x}_3 = -\beta x_2, \end{cases}$$

$$g(x_1) = \begin{cases} \dfrac{b\pi}{2a}(x_1 - 2ac), & \text{if} \ x_1 \geq 2ac, \\ -b \sin\left(\dfrac{\pi x_1}{2a} + d\right), & \text{if} \ -2ac < x_1 < 2ac, \\ \dfrac{b\pi}{2a}(x_1 + 2ac), & \text{if} \ x_1 \leq -2ac, \end{cases} \tag{4}$$

an 8-scroll attractor is produced for the parameters $\alpha = 10.814$, $\beta = 14, a = 1.3, b = 0.11, c = 7, d = 0$. The system (4) admits the unstable origin equilibrium point $E_0 = (0, 0, 0)$ and the following unstable equilibrium points $E_{\pm1} = (\pm2.6, 0, \mp2.6)$, $E_{\pm2} = (\pm5.2, 0, \mp5.2)$, $E_{\pm3} = (\pm7.8, 0, \mp7.8)$, $E_{\pm4} = (\pm10.4, 0, \mp10.4)$, $E_{\pm5} = (\pm13, 0, \mp13)$, $E_{\pm6} = (\pm15.6, 0, \mp15.6)$, $E_{\pm1} = (\pm18.2, 0, \mp18.2)$.

According to (2), this system can be represented by a T-S fuzzy model as follows:

$$\begin{aligned} &R^1: \text{IF} \ x_1 \ \text{is} \ M_1 \ \text{THEN} \ \dot{x}(t) = A_1 x(t), \\ &R^2: \text{IF} \ x_1 \ \text{is} \ M_2 \ \text{THEN} \ \dot{x}(t) = A_2 x(t), \end{aligned} \tag{5}$$

where $x(t) = [x_1(t), x_2(t), x_3(t)]^T$, $A_1 = \begin{bmatrix} \alpha D & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$, $A_2 = \begin{bmatrix} -\alpha D & \alpha & 0 \\ 1 & -1 & 1 \\ 0 & -\beta & 0 \end{bmatrix}$, $M_1(x_1) = \frac{1}{2}\left(1 + \frac{g(x_1)}{D}\right)$ and $M_2(x_1) = \frac{1}{2}\left(1 - \frac{g(x_1)}{D}\right)$, with $D = 4$. Fig. 1 shows the T-S fuzzy model of the 8-scroll Chua's circuit.

2.2. Fuzzy modeling of the multi-scroll Chen system

For the multi-scroll Chen system [4]

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), \\ \dot{x}_2 = (c - a)x_1 - x_1 x_3 + c x_2 + d x_1 \sin x_3, \\ \dot{x}_3 = x_1 x_2 - b x_3, \end{cases} \tag{6}$$

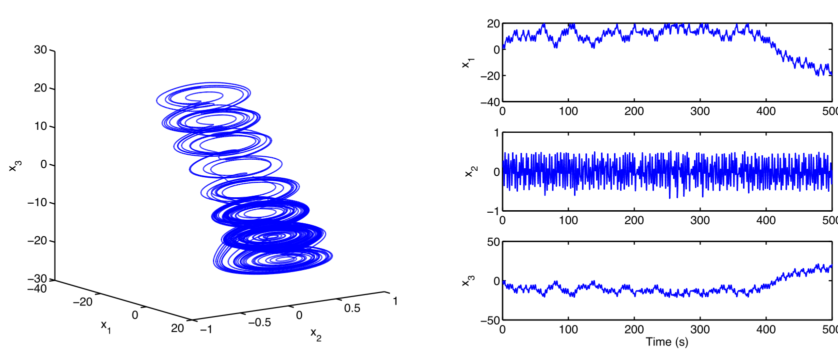
a 6-scroll attractor is produced for the parameters $a = 35, b = 3, c = 28$ and $d = 8$. The system (4) admits the origin as an unstable equilibrium point $E_0 = (0, 0, 0)$, and ten unstable equilibrium points given by $E_{\pm1} = (\pm6.999, \mp6.999, 16.331)$, $E_{\pm2} = (\pm7.457, \mp7.457, 18.536)$, $E_{\pm3} = (\pm8.102, \mp8.102, 21.880)$, $E_{\pm4} = (\pm8.792, \mp8.792, 25.771)$, $E_{\pm5} = (\pm9.059, \mp9.059, 27.356)$.

This system is exactly represented by a T-S fuzzy model with the following four IF-THEN fuzzy rules

$$\begin{aligned} &R^1: \text{IF} \ -x_3 + d\sin x_3 \ \text{is} \ M_{11} \ \text{THEN} \ \dot{x}_2 = (c - a)x_1 + x_1 D_1 \\ &R^2: \text{IF} \ -x_3 + d\sin x_3 \ \text{is} \ M_{12} \ \text{THEN} \ \dot{x}_2 = (c - a)x_1 - x_1 D_1 + c x_2 \\ &R^3: \text{IF} \ x_1 \ \text{is} \ M_{21} \ \text{THEN} \ \dot{x}_3 = x_2 D_2 - b x_3 \\ &R^4: \text{IF} \ x_1 \ \text{is} \ M_{22} \ \text{THEN} \ \dot{x}_3 = -x_2 D_2 - b x_3 \end{aligned} \tag{7}$$

where $x(t) = [x_1(t), x_2(t), x_3(t)]^T$,
$A_1 = \begin{bmatrix} -a & a & 0 \\ c - a + D_1 & c & 0 \\ 0 & D_2 & -b \end{bmatrix}$, $A_2 = \begin{bmatrix} -a & a & 0 \\ c - a - D_1 & c & 0 \\ 0 & -D_2 & -b \end{bmatrix}$,
$M_{11}(x_3) = \frac{1}{2}\left(1 + \frac{-x_3 + d \ \sin \ x_3}{D_1}\right)$, $M_{12}(x_3) = \frac{1}{2}\left(1 - \frac{-x_3 + d \ \sin \ x_3}{D_1}\right)$,
$M_{21}(x_1) = \frac{1}{2}\left(1 + \frac{x_1}{D_2}\right)$ and $M_{22}(x_1) = \frac{1}{2}\left(1 - \frac{-x_1}{D_2}\right)$, with $D_1 = 50$ and $D_2 = 23$. Fig. 2 illustrates the T-S fuzzy model of the multi-scroll Chen system.

To support our analysis to be carried out in the following sections, some existing results are needed.



(a) (b)

Fig. 1. T-S fuzzy model of the 8-scroll Chua's circuit. (a) Phase space. (b) Time evolution of the state variables.