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Stochastic fast smooth second-order sliding modes terminal guidance law design

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ABSTRACT

Aiming at handling the track imprecision caused by inertial lag, model uncertainties and atmospheric environment disturbances, as well as stochastic noises, a terminal guidance law based on stochastic fast smooth second-order sliding modes control theory is proposed. This paper considers targets performing evasive maneuvers and develops a high-order sliding mode observer. A concept of finite-time mean-square practical convergence, considering the non-equilibrium additive noise of the guidance system, is presented. And according to this concept, the finite-time convergent guidance law is deduced. The feasibility of the new guidance law is exemplified through computer simulations and the guidance performance is compared with augmented proportional navigation guidance law, sliding mode guidance law and nonsingular terminal sliding mode guidance law.

1. Introduction

An important criterion of a homing missile is the tracking accuracy, which is closely related to guidance, navigation, and control crucially [1]. To achieve higher guidance precision against ever-increasing performance targets, the line-of-sight (LOS) rate needs converge to zero fast, which makes the terminal trajectory straight and the normal acceleration of the missile small. However, due to the missile seeker detection lag, guidance update rate limitation, missile rudder inertial delay, model uncertainties and atmospheric environment disturbances [2], the LOS rate cannot be converge to zero within a short time.

A series of sliding mode control (SMC) algorithms has being devoted to design the homing missile guidance law due to its advantages of handling bounded uncertainties, disturbances and unmolded dynamics [3]. The SMC guidance law achieved smaller acceleration ratio compared to traditional proportional navigation (PN) and augmented proportional navigation (APN) guidance laws [4,5]. However, classical SMC cannot ensure the LOS rate converge to zero in finite-time [6]. So Zhou presented a new guidance law based on SMC that can guarantee the LOS rate converge to zero or its small neighborhood in finite-time [7]. Whereas, the guidance law designed in the framework of first-order SMC requires the system relative degree equal to 1 with respect to the sliding variable and the controller yields a heavy chattering [8–10]. And in Ref. [8–10], to deal with the intrinsic difficulties of classical SMC, the high-order sliding mode (HOSM) controllers are presented. A new smooth second-order sliding mode (SSOSM) control driven by uncertain sufficiently smooth disturbances is proposed and proved by Shtessel [11,12]. The main limitations of this guidance law are the simplification of the state noise and the dependences of the perfect knowledge of the range to target and the range rate, which is usually hard to get an accuracy value. Another limitation of this method is the specific requirement of the target normal acceleration.

In order to solve the defects in previous research, a novel stochastic fast smooth second-order sliding mode (SFS-SOSM) method with a finite-time convergence in the presence of evasive target maneuvers, uncertainties, disturbances and stochastic noise, is proposed in this paper. A new concept of finite-time mean-square practical (FTMSP) stability is introduced to investigate the finite-time convergence of the stochastic sliding surface and the FTMSP convergence of SFS-SOSM control is proved by Itô's formula.

The paper is organized as follows: Section 2 states the missile-target engagement kinematics. The SFS-SOSM control algorithm is derived and its FTMSP convergence is proved in Section 3. In Section 4, a smooth guidance law based on SFS-SOSM is presented and its performance is verified via computer simulations compared with augmented proportional navigation guidance law (APN), sliding mode guidance law (SMG) and nonsingular terminal sliding mode guidance law (NT-SMG) in Section 5.

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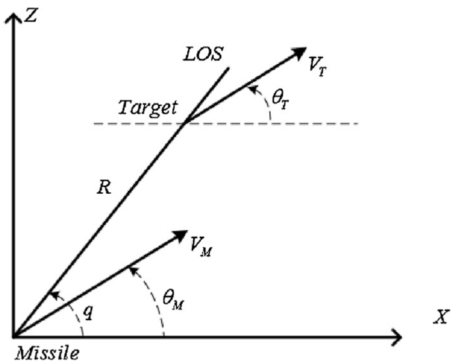

Fig. 1. Typical planar engagement geometry.

2. Planar engagement model and intercept strategy

2.1. Problem formulation

Consider the planar homing case that the missile moves within the vertical plane, a typical engagement scenario is presented in Fig. 1.

The planar missile-target engagement kinematics can be easily derived as

$$\begin{cases} \dot{r} = V_T\cos(q-\theta_T) - V_M\cos(q-\theta_M) \\ r\dot{q} = -V_T\sin(q-\theta_T) + V_M\sin(q-\theta_M) \end{cases} \tag{1}$$

where q is the LOS angle, r is the range along LOS, V_T and V_M are target velocity and missile velocity, θ_T and θ_M are target aspect angle and missile lead angle.

The eq. (1) can be reduced by differentiating both sides of the second equation with respect to time and substituting the first equation into it leads to the following equation

$$\ddot{q} = -\frac{2\dot{r}}{r}\dot{q} - \frac{1}{r}a_M + \frac{1}{r}a_T \tag{2}$$

where a_M is missile normal acceleration as a control input, a_T is target normal acceleration that is considered as unknown bounded disturbance. Denote $\omega = \dot{q}$ as the LOS rate, Eq. (2) becomes the following equation

$$\dot{\omega}(t) = -\frac{2\dot{r}(t)}{r(t)}\omega(t) - \frac{1}{r(t)}a_M + \frac{1}{r(t)}a_T \tag{3}$$

where the starting time of the guidance process is taken to be zero, ω is an uncertain sufficiently smooth function. Assume that the state noise $\xi(t)$ is a zero-mean white Gaussian process with covariance $Q(t)$, Eq. (3) can be rewritten as

$$\dot{\omega}(t) = -\frac{2\dot{r}(t)}{r(t)}\omega(t) - \frac{1}{r(t)}a_M + \frac{1}{r(t)}a_T + \xi(t) \tag{4}$$

2.2. Intercept strategy

It is well known that in the space interception where a missile is intercepting a target with maneuverability, the time of terminal guidance is only several seconds such that the guidance law is required to ensure finite time convergence of the LOS angular rate [7,13]. To ensure finite time convergence of the LOS angular rate, the guidance law is derived to stabilize the system (4) on the manifold

$$\sigma = \omega(t) + \rho\int_0^t \omega(\tau)d\tau \tag{5}$$

where $\rho = const.>0$.

Differentiating both sides of Eq. (5) with respect to time, we arrive at the following equation

$$\dot{\sigma} = \dot{\omega} + \rho\omega = \left(\rho - \frac{2\dot{r}}{r}\right)\omega - \frac{1}{r}a_M + \frac{1}{r}a_T + \xi \tag{6}$$

The guidance command can be obtained by employ SFSSOSM control, which is derived and analyzed in the next section.

3. Stochastic fast smooth second-order sliding mode control

It is obvious that system (6) is driven by additive noise, meaning that the equation doesn't have any equilibrium [15]. Instead of convergence to the origin, the more reasonable way is to stabilize σ to a small neighborhood of zero in finite time [7,14]. Practical stability, proposed by La Salle and Lefschetz [16], is motivated by the fact that the state of a physical system may be mathematically unstable, but it operates sufficiently near the desired state. With the aid of this concept, a new concept of FTMSP convergence is introduced first, and then the SFS-SOSM control is derived and its FTMSP convergence is proved in this section.

3.1. Finite-time mean-square practical stability

The definition of practical stability given in [14] is extended to a stochastic nonlinear system as follows.

Definition (FTMSP convergence): Denote $x(t)$ the solution process of system (4) under the initial condition $x(t_0) = x_0$. The sliding surface $\sigma = \sigma(x(t)) = 0$ is called finite-time mean-square practical (FTMSP) convergent if, given real number pair δ, $\varepsilon > 0$ satisfying certain conditions, there exists a finite setting time $T \geq 0$, which is dependent on x_0, such that $E\left\|\sigma(t_0)\right\|^2 \leq \delta$ implies $E\left\|\sigma(t)\right\|^2 \leq \varepsilon$ for any $t - t_0 > T$.

It follows from the above definition that the $E\left\|\sigma(t)\right\|^2$ is sufficiently close to zero in finite-time if the sliding surface is FTMSP convergent.

3.2. Prescribed sliding variable dynamics

On the ground of the smooth second-order sliding mode (SOSM) control proposed by Shtessel [11], an extended stochastic fast smooth SOSM (SFS-SOSM) control can be deprived and the dynamics of the sliding variable σ is designed to have the following form:

$$\begin{cases} \dot{\mu}_1 = -k_1|\mu_1|^{(m-1)/m}sgn(\mu_1) - k_2\mu_1 - k_3|\mu_2|sgn(\mu_1) + (\omega - z_1) + \xi \\ \dot{\mu}_2 = -k_4|\mu_1|^{(m-2)/m}sgn(\mu_2) - k_5\mu_2 \end{cases} \tag{7}$$

where $\mu_1 = \sigma$, $m = const.>2$, $k_i = const.>0$ $(i = 1, 2, 3, 4, 5)$, ξ is the noise signal mentioned in Eq. (4).

Let $\boldsymbol{\mu} = [\mu_1, \mu_2]^T$, then Eq. (7) is a stochastic system with respect to state $\boldsymbol{\mu}$ and can be represented as

$$d\boldsymbol{\mu} = \boldsymbol{f}(\boldsymbol{\mu})dt + \boldsymbol{g}dW(t) \tag{8}$$

where $W(t)$ is a 1-dimensional standard Brownian motion and $\boldsymbol{f}$, $\boldsymbol{g}$ are

$$\begin{aligned} \boldsymbol{f}(\boldsymbol{\mu}) &= -k_1\left|\mu_1\right|^{(m-1)/m}sgn(\mu_1) - k_2\mu_1 - k_3\left|\mu_2\right|sgn(\mu_1) \\ &\quad -k_4\left|\mu_1\right|^{(m-2/m)}sgn(\mu_2) - k_5\mu_2 \\ \boldsymbol{g} &= \begin{bmatrix} \sqrt{\boldsymbol{Q}} \\ 0 \end{bmatrix} \end{aligned} \tag{9}$$

System (8) is a stochastic nonlinear system with additive noise. Hereafter, FTMSP convergence is employed to analyze the finite-time convergence of system (8).