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A new four-scroll chaotic attractor and its engineering applications

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ABSTRACT

In this paper we investigate a new system of three coupled nonlinear ordinary differential equations, whose dynamics support periodic and chaotic attractors as certain parameters vary. In its most general form, the system has nine parameters. However we can set up to three of these parameters to zero, and still obtain complex dynamics. Here we discuss the case where only two of these parameters are set to zero, and present two-parameter bifurcation linear stability curves for various combinations of the remaining parameters. Then we compute Lyapunov exponents, to verify the regimes of chaotic dynamics, and use adaptive control theory to influence the behaviour. An electronic circuit model of the new chaotic system has been designed and its simulations have been performed using an ORCAD-PSpice program. An experimental realisation of the new chaotic circuit has been carried out and oscilloscope outputs have been compared with numerical (digital) and electronic circuit simulation results. We then used the chaotic system to design a random number generator, and show that the new system has the potential of being used in several scientific and engineering fields such as communication, image processing, physics and mechatronics.

1. Introduction

Over recent years, new and different chaotic and hyper chaotic systems with interesting features and with the potential of being practically used in several fields have been introduced to the literature [1–10]. Various methods have been used for the encryption of data [11–15]. In this paper we investigate one such system of three coupled nonlinear ordinary differential equations, whose dynamics support periodic and chaotic attractors as certain parameters vary. In its most general form, the system has nine parameters. However we can set up to three of these parameters to zero, and still obtain complex dynamics. Here we discuss the case where only two of these parameters are set to zero, and present two-parameter bifurcation linear stability curves for various combinations of the remaining parameters. We also compute Lyapunov exponents to verify the regimes of chaotic dynamics, and use adaptive control theory to influence the behaviour.

Such a chaotic system can be used as a random number generator (RNG). In some studies, encryption has been carried out by generating random numbers from chaotic systems and using these numbers as keys. Randomness of the generated numbers directly affects the reliability in encryption practices.

There are many studies in the literature on generating random numbers [16–19]. One conclusion from such studies in the literature is there are intensive investigations of chaotic systems in the context of encryption on multimedia data with different methods.

A significant deficit in chaotic RNG based encryption studies is that randomness test results of FIPS-140-1 and NIST-800-22 have not been included and security analyses regarding encrypted data are insufficient. We rectify these omissions her and perform security analyses: namely, correlation, histogram, key sensitivity and key length analysis on the new chaotic attractor.

The paper is organised as follows. In Section 2 we introduce the most general form of the new chaotic system, and perform a linear stability analysis. In Section 3 we investigate a simplified version in which two of the parameters are set to zero. Section 4 is concerned with adaptive control of the general form of the system, and in Section 5 we determine Lyapunov exponents, and introduce the OrCAD-PSpice formulation, together with the electronic circuit schema of the system. The remaining Sections 6–8 deal with encryption and random number generation with the new chaotic system.

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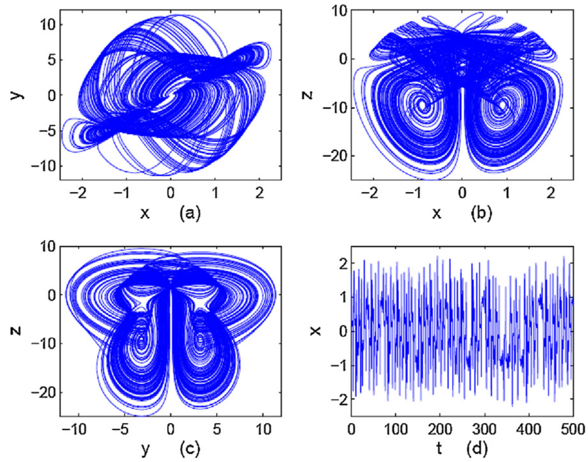

Fig. 1. Phase portraits in the (x, y), (x, z), (y, z) planes and the time series for $x(t)$ for the parameter values in (2).

2. The general system

In its most general form, we introduce the nonlinear autonomous system

$$\dot{x} = p_1 y - p_2 x + p_3 xz, \tag{1a}$$

$$\dot{y} = -p_4 xz - p_5 x + p_6 yz + p_7 x, \tag{1b}$$

$$\dot{z} = p_8 - p_9 y^2. \tag{1c}$$

For the parameter choice

$$p_1 = 1, \quad p_2 = 0.7, \quad p_3 = 0.3, \tag{2a}$$

$$p_4 = 4, \quad p_5 = 4.4, \quad p_6 = 1, \quad p_7 = 0.1, \tag{2b}$$

$$p_8 = 10.0, \quad p_9 = 1.0, \tag{2c}$$

we observe the chaotic behaviour, shown in Fig. 1. Note that equations (1) are invariant under the reflectional symmetry $(x, y, z) \to (-x, -y, z)$.

2.1. Linear stability analysis

The system (1) admits the equilibrium states

$$(x, y, z) = (X_{e\pm}, Y_e, Z_{e\pm}) \tag{3}$$

where

$$Y_e = \pm\sqrt{\frac{p_8}{p_9}}; \tag{4}$$

$$X_{e+} = \frac{p_1 Y_e}{p_2 - p_3 Z_{e+}}, \quad X_{e-} = \frac{p_1 Y_e}{p_2 - p_3 Z_{e-}}; \tag{5}$$

and

$$Z_{e\pm} = \frac{p_2 p_6 - p_1 p_4}{2p_3 p_6} + \frac{\sqrt{disc(Z_e)}}{2p_3 p_6}, \tag{6a}$$

$$Z_{e-} = \frac{p_2 p_6 - p_1 p_4}{2p_3 p_6} - \frac{\sqrt{disc(Z_e)}}{2p_3 p_6}, \tag{6b}$$

where

$$discZ_e = (p_2 p_6 - p_1 p_4)^2 - 4p_3 p_6 p_1 (p_5 - p_7). \tag{7}$$

Because of the reflectional symmetry, we can fix the sign of Y_e as either + or − in the following analysis. The linear stability of (3)–(6) is determined by solving the cubic characteristic equation:

$$\Lambda^3 + A_2 \Lambda^2 + A_1 \Lambda + A_0 = 0, \tag{8}$$

where

$$A_2 = p_2 - Z_e(p_3 + p_6), \tag{9a}$$

$$A_1 = \frac{2p_9 X_e Y_e (p_5 - p_7)}{Z_e}, \tag{9b}$$

$$A_0 = \frac{2p_8 [p_1(p_5 - p_7) - p_3 p_6 {Z_e}^2]}{Z_e}. \tag{9c}$$

From (7), we have a saddle-node bifurcation when $discZ_e = 0$. For $discZ_e < 0$, there are no fixed points, while for $discZ_e > 0$, there are two fixed points. We must therefore impose the condition $discZ_e \geq 0$ on the parameters p_j in order to continue with our analysis. Since Z_e appears in a saddle-node bifurcation, one of the fixed points in (6) will be stable and the other unstable.

It is possible to reduce the number of parameters p_j, and still have chaotic behaviour of the form, shown in Fig. 1. We integrated system (1) numerically to produce a series of bifurcation transition diagrams, by fixing each parameter in turn from (2), and noting whether we still obtained chaotic behaviour when that particular parameter was zero. This procedure enables us to set up to three parameters to zero: $p_2 = 0$, $p_5 = 0$ and $p_7 = 0$. However we chose only to set $p_2 = 0$ and $p_5 = 0$ to zero, since a non-zero p_7 can support Hopf bifurcations. This is the system that we study in the next section of the paper.

3. The simplified system

Setting $p_2 = 0 = p_5$, (1) becomes

$$\dot{x} = p_1 y + p_3 xz, \tag{10a}$$

$$\dot{y} = -p_4 xz + p_6 yz + p_7 x, \tag{10b}$$

$$\dot{z} = p_8 - p_9 y^2. \tag{10c}$$

The fixed point Y_e is still given by (4), while (5) becomes

$$(X_{e+}, X_{e-}) = -\frac{p_1 Y_e}{p_3} \left(\frac{1}{Z_{e+}}, \frac{1}{Z_{e-}} \right), \tag{11}$$

and Z_e satisfies the quadratic equation:

$$p_3 p_6 {Z_e}^2 + p_1 p_4 Z_e - p_1 p_7 = 0. \tag{12}$$

Since all the parameters $p_j > 0$ in (2), when $p_2 = p_5 = 0$, (6) shows that Z_{e+} and Z_{e-} always exist. However, this may no longer be the case if any of these parameters becomes negative. The cubic characteristic Eqs. (7) and (8) also simplifies to give:

$$\Lambda^3 + B_2 \Lambda^2 + B_1 \Lambda + B_0 = 0, \tag{13}$$

where

$$B_2 = -Z_e(p_3 + p_6), \tag{14a}$$

$$B_1 = \frac{-2p_7 p_9 X_e Y_e}{Z_e}, \tag{14b}$$

$$B_0 = \frac{2p_1 p_8 (p_4 Z_e - 2p_7)}{Z_e}, \tag{14c}$$

and we have used (12) to eliminate ${Z_e}^2$.

3.1. Bifurcations

A steady state bifurcation occurs when $\Lambda = 0$. From (14), this corresponds to $B_0 = 0$, and implies that either $p_1 = 0$ or $p_8 = 0$ or $Z_e = 2p_7/p_4$. Substituting this third equality into (12) gives $4p_3 p_6 p_7 + {p_4}^2 p_1 = 0$, which cannot be satisfied unless one of the parameters $p_i < 0$ for some i.

A Hopf bifurcation occurs when $\Lambda = \pm \imath\Omega$, where Ω satisfies

$$\Omega^2 = B_1 = \frac{B_0}{B_2} > 0. \tag{15}$$