Optik 127 (2016) 5506–5514

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Reduced-order synchronization of time-delay chaotic systems with known and unknown parameters

Israr Ahmad [a,b], Azizan Bin Saaban [a], Adyda Binti Ibrahim [a], Mohammad Shahzad [b], M. Mossa Al-sawalha [c,*]

[a] School of Quantitative Sciences, College of Arts & Sciences, University Utara Malaysia, Malaysia
[b] College of Applied Sciences Nizwa, Ministry of Higher Education, Oman
[c] Mathematics Department, Faculty of Science, University of Hail, Saudi Arabia

ARTICLE INFO

Article history:
Received 23 January 2016
Accepted 29 February 2016

Keywords:
Reduced-order synchronization
Lyapunov–Krasovskii functional theory
Nonlinear control
Adaptive control
Time-delay chaotic systems

ABSTRACT

In this article, a novel robust nonlinear controller approach is focused to study theoretically, the reduced-order synchronization phenomena of two unrelated time-delayed chaotic systems under the determined and unknown parameters. It is assumed that the two systems are perturbed by the bounded unstructured uncertainties and unknown external disturbance. Based on the Lyapunov–Krasovskii functional theory, a robust nonlinear synchronization controller is focused and a suitable Lyapunov functional is constructed so that they establish the globally asymptotical stability of the closed-loop at the origin. Subsequently, suitable adaptive laws of unknown parameters are designed to identify the unknown parameters. Finally, the effectiveness of the proposed reduced-order synchronization approach is verified by numerical simulations. A brief comparison of the present study with prior works has been given.

1. Introduction

The study presented by Pecorra and Carroll [1] gave a criterion for the complete synchronization of two nearly identical chaotic systems. Since, then, synchronization research has rapidly developed owing to several potential applications [2–4]. For the last two decades, synchronization of chaotic systems has widely been addressed in different scientific disciplines on the basis of theoretical investigations and experimental studies [5,6]. Subsequently, several effective chaos based synchronization control techniques and methods have been developed. These include, adaptive control [6], sliding mode control [7], linear active control [8], linear feedback control [9], the hybrid projective synchronization [10], backstepping method [11] and the lag synchronization [12], etc.

Till now, there are two types of synchronization has been discussed, namely, synchronization between chaotic systems with the same orders and synchronization between chaotic systems with different orders. In the latter case, synchronization can be further classified into two basic categories termed as increased-order synchronization and reduced-order synchronization (ROS) [13]. In ROS scheme, the order of the controlled slave/response is less than that of the master/drive system such that the slave system is forced to track the geometrical properties of the projection part of the master system [14]. The ROS phenomena can be observed in many physical and natural systems due to the fact that the components of certain complex systems cannot supposed to be identical or of the same orders [15]. The ROS have potential applications in physical, social, information processing, and biological systems [16]. The ROS of chaotic systems has been investigated by some researchers [14–19] (cited here among others).

The effect of time-delay on nonlinear dynamical systems is quite noteworthy. The memory effect and the finite signal transmission speeds are the bases of time-delays [20]. Time-delays chaotic/hyperchaotic systems exhibit multistability and can produce more complex dynamics that enhances the security of the transmitted data. The synchronization of time-delay chaotic systems has attracted a great interest in the literature concerned [21,22]. Because of the signal propagation delays, it is not reasonable to require the slave system to synchronize the master system at the same time [22]. Thus, the time-delay plays an important role in chaotic synchronization. To the best

* Corresponding author. Tel.: +966 543156861.
E-mail addresses: iak_2000plus@yahoo.com (I. Ahmad), azizan.s@uum.edu.my (A.B. Saaban), adyda@uum.edu.my (A.B. Ibrahim), dmsinfinite@gmail.com (M. Shahzad), sawalha_moh@yahoo.com (M. Mossa Al-sawalha).

http://dx.doi.org/10.1016/j.ijleo.2016.02.078

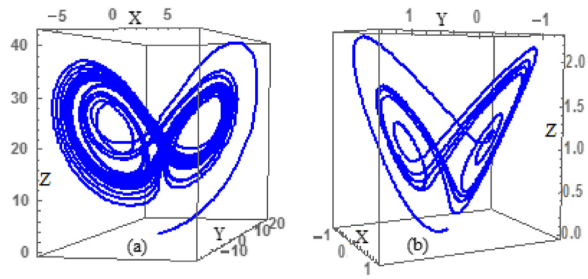

Fig. 1. 3D phase portrait of the (a) LSH system and, (b) SM chaotic system.

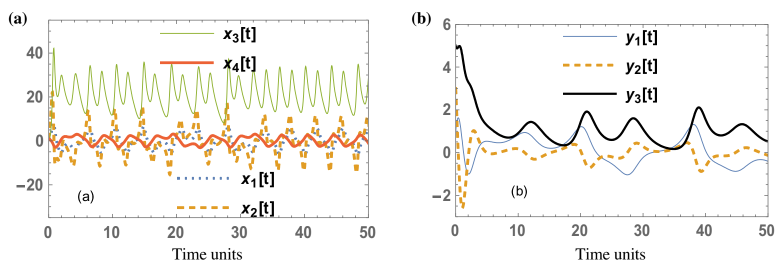

Fig. 2. (a) Time series of the chaotic trajectories (LSH). (b) Time series of the chaotic trajectories (SM).

of the authors' knowledge, no attempt has been made for the ROS of time-delay chaotic systems because of computational and structural difficulties in the controller design and this has remained as an open problem.

To deal with the stability analysis and less conservative conditions in combination with different kinds of perturbations and time-delays, the authors present a novel contribution to this line. Based on the Lyapunov–Krasovskii functional theory [23], a nonlinear feedback controller is addressed and sufficient algebraic conditions are derived to achieve the globally asymptotical ROS of time-delay chaotic systems with the presence of unstructured uncertainties and unknown external disturbance under the determined parameters. Furthermore, in practical applications, some or all of the system's parameters cannot be exactly known in advance. Therefore, to tackle the analytical complications produced by the parameters uncertainties, the adaptive based synchronization controller and analytic expression for the controller and adaptive laws of unknown parameters are developed. Numerical experimental results depicted graphically show that the proposed control strategy is robust against different kinds of perturbations and the time-delay. The unknown parameters with different numerical values are identified with high accuracy. Thus, the proposed controller design is effective, reliable, and convenient to implement for real world applications.

The outline of the paper is as follows. In Section 2, description of the time delay Lorenz–Stenflo hyperchaotic (LSH) [24] and time-delay Shimizu–Morioka (SM) [25] chaotic systems are given and solved the problem of ROS between the time-delay LSH and time delay SM systems under the determined parameters. In Section 3, the adaptive control strategy for the ROS between the time-delay unknown LSH and time-delay unknown SM chaotic systems are obtained. Finally, the concluding remarks are given in Section 4.

2. Reduced-order synchronization between the time-delay Lorenz–Stenflo hyperchaotic and time-delay Shimizu–Morioka chaotic systems under the determined parameters

2.1. Description of the Lorenz–Stenflo hyperchaotic system

Stenflo [25] proposed and investigated a new hyperchaotic system which is now known as the famous Lorenz–Stenflo hyperchaotic system. The new hyperchaotic attractor was basically coined by adding a fourth variable and a new control parameter d to the original Lorenz system. The LSH system was basically derived from measuring the low-frequency, short wavelength wave perturbation in Acoustic-Gravity waves in the atmosphere. The proposed model also has been used to describe high-latitude phenomena [26]. The mathematical model of the LSH system is described as follows:

$$\left.\begin{aligned} \dot{x}_1(t) &= a(x_2(t)-x_1(t)) + bx_4(t) \\ \dot{x}_2(t) &= cx_1(t) - x_2(t) - x_1(t)x_3(t) \\ \dot{x}_3(t) &= -dx_3(t) + x_1(t)x_2(t) \\ \dot{x}_4(t) &= -x_1(t) - ax_4(t) \end{aligned}\right\}, \tag{1}$$

where $[x_1(t), x_2(t), x_3(t), x_4(t)]^T \in R^4$, are the state vectors and $a > 0$, $b > 0$, $c > 0$, $d > 0$, are the control parameters of the system that represents the Rayleigh number, Prandtl number, rotation number and geometric parameter, alternatively. The LSH system governed by system (1) shows a complex dynamical behavior for the parameters values $a = 1$, $b = 1.5$, $c = 26$, $d = 0.7$, as shown in Figs. 1(a) and 2(b).