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# Multi-objective quantum-behaved particle swarm optimization algorithm with double-potential well and share-learning

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ABSTRACT

For improving the convergence accuracy and diversity of multi-objective optimization algorithm a multi-objective quantum-behaved particle swarm optimization algorithm with double-potential well and share-learning is proposed, which overcomes the deficiency of particles readily gathering in identical solutions. The two local attractors, inside and outside, are introduced to construct the particle locations updating model, using the quantum tunneling and transition effects in double-potential well model. In this way, the particle moves to the solution sparseness region in later evolution stage, so as to avoid gathering in the single local attractor and escape from local optimum. Therefore the optimization accuracy of the algorithm is improved. The share-learning strategy is adopted to extend the search range of particles and increase the diversity of solutions. The problem of easily converging to boundary solutions in quantum-behaved particle swarm optimization algorithm could be avoided. Simulation results show that the proposed algorithm makes excellent performance in optimization accuracy, convergence, diversity, and distribution, compared with three existing algorithms. Moreover, the proposed algorithm can hold on better convergence and distribution performance when handling high-dimensional multi-objective problems.



## 1. Introduction

The multi-objective optimization (MOO) problems widely exist in the realms of path planning, control system design, structure optimization, and operational scheduling. Generally, these objectives are conflicting with each other, which means that an improving of one objective may result in performance degradation of another objective. Therefore, we must make a compromise among these objectives since it is impossible for all these objectives to attain the optimum. As an intelligent optimization method, the evolutionary algorithm becomes a major branch to handle the multi-objective optimization problem [1,2]. The particle swarm optimization (PSO) algorithm [3,4], one of the evolutionary algorithms, is widely applied to MOO problem due to the merits of fast convergence, simple operation and less parameters. In 2002, Coello [5] first proposed the multi-objective particle swarm optimization (MOPSO) algorithm, and then the adaptive grid was applied to maintain external file [6]. Raquel [7] adopted a new strategy named crowded distance sorting to update the external file of particle swarm. Lechuga [8] improved the diversity of solution via niche technique. And Coelho [9] introduced the Gauss mutation operator to MOPSO algorithm for improving the convergence performance.

In the standard PSO system, the convergence of particles is implemented by trajectories, and they could not search the entire solution space due to the shortcoming of low speed. Bergh [10] demonstrated that the standard PSO algorithm cannot converge to global optimal solution when solving complicated optimization problem. Therefore it is an essential flaw for PSO algorithm with low precision in local searching. For this problem, Sun [11,12] proposed the quantum-behaved particle swarm optimization (QPSO) algorithm, which was inspired by the quantum mechanics theory. In QPSO, a new model for the particle location update was built via the bound state of quantum $\delta$ potential well. And the global convergence of QPSO was proved by Markov process and probabilistic metric space, respectively [13,14]. Since QPSO algorithm displays excellent performance on single objective optimization [15,16], Shi [17] applied it to MOO problem and compared with multi-objective PSO (MOPSO) algorithm, which showed the superiority of multi-objective QPSO (MOQPSO) on the convergence speed and accuracy. The MOQPSO algorithm has not been widely adopted because the faster convergence rate of QPSO may result in premature

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convergence, which reduces the population diversity and convergence precision. Though this disadvantage could be mended via introducing the Gaussian mutation operator and crowded distance sorting, the deficiency that the particles tend to accumulate at some special locations does not change essentially.

Aiming at improving the accuracy, diversity and uniformity of QPSO for handling MOO problem, this paper proposes a new MOQPSO algorithm based on double-potential well and share-learning (MOQPSO-DPS). According to the quantum tunneling and transition characteristics of double potential well, two attractors inside and outside are introduced to construct the model of particle location update, so that the particles could transfer to distributed sparse region of solution at later evolution process. In addition, the share-learning strategy is adopted to expand the search scopes. Moreover, the Gaussian mutation operator is also applied in the algorithm so as to improve the local searching precision, which prompts the particles to seek out the real Pareto optimal solution sets.

## 2. Basic concept of multi-objective optimization

Usually the basic concepts in multi-objective optimization field are as follows [18].

*MOO problem*: The model of MOO problem can be described as (without loss of generality, the paper supposes that it is a minimization problem):

$$\begin{aligned} &\min\ \boldsymbol{y} = f(\boldsymbol{x}) = [f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x})] \\ &\text{s.t.}\quad g_i(\boldsymbol{x}) \le 0,\ i = 1, 2, \cdots, p \\ &\qquad h_i(\boldsymbol{x}) = 0, \quad i = 1, 2, \cdots, q \end{aligned} \tag{1}$$

where $\boldsymbol{x} \in D$ is the decision vector and $\boldsymbol{y} \in Y$ is the objective vector. $f_i(\boldsymbol{x})$ are objective functions. $D$ and $Y$ are the decision space and objective space. $g_i(\boldsymbol{x}) \le 0$ and $h_i(\boldsymbol{x}) = 0$ are inequality and equality constraints, respectively.

*Pareto dominance*: For two given points $\boldsymbol{x}^0$, $\boldsymbol{x}^1$ and, $\boldsymbol{x}^0$ Pareto dominates $\boldsymbol{x}^1$, written as $\boldsymbol{x}^0 \prec \boldsymbol{x}^1$, if and only if

$$\begin{aligned} &\forall i \in \left\{1, 2, \cdots, m\right\} : f_i(\boldsymbol{x}^0) \le f_i(\boldsymbol{x}^1),\ \text{and} \\ &\exists j \in \left\{1, 2, \cdots, m\right\} : f_i(\boldsymbol{x}^0) < f_i(\boldsymbol{x}^1) \end{aligned} \tag{2}$$

*Pareto optimal*: The solution $\boldsymbol{x}^0$ is Pareto optimal, if and only if $\neg \exists x^1 : x^1 \prec x^0$.

*Pareto optimal set*: The set $P_S = \left\{\boldsymbol{x}^0 | \neg\exists \boldsymbol{x}^1 : \boldsymbol{x}^1 \prec \boldsymbol{x}^0\right\}$ constituted by all of the Pareto optimal solutions.

*Pareto optimal front*: The region constituted by the objective function values of all the Pareto optimal solutions.

$$P_F = \left\{\boldsymbol{F}(\boldsymbol{x}) = (f_1(\boldsymbol{x}), f_2(\boldsymbol{x}), \cdots, f_m(\boldsymbol{x}))\,|\boldsymbol{x} \in P_S\right\}$$

## 3. Multi-objective quantum-behaved particle swarm optimization algorithm

### 3.1. Quantum-behaved particle swarm optimization algorithm

Refs. [11,12] proposed a quantum-behaved particle swarm optimization algorithm based on quantum mechanics theory, using the quantum $\delta$ potential well. It describes the aggregation of particles via the bound state of quantum, and the particles in bound state could appear at any location of the solution space at a certain probability. Therefore, the particle can search the entire feasible solution space at each iteration, which is the most striking difference between QPSO and PSO.

For the standard PSO algorithm, the particle state is determined by location and speed. Yet, the particle of QPSO algorithm only updates the location without speed. Ref. [12] derived the particle location update equation of QPSO by wave function of $\delta$ potential well, that is

$$\psi(Y) = \frac{1}{\sqrt{L}} e^{-|Y|/L} \tag{3}$$

Its probability density function $Q$ and distribution function $F$ can be written as

$$\begin{aligned} &Q(\boldsymbol{Y}) = \left|\psi(\boldsymbol{Y})\right|^2 = \frac{1}{\boldsymbol{L}} e^{-2|Y|/L} \\ &\boldsymbol{F}(\boldsymbol{Y}) = e^{-2|Y|/L} \end{aligned} \tag{4}$$

Pick a uniform random number $u$ in [0,1], and set $u = F(\boldsymbol{Y})$, then we get

$$\boldsymbol{Y} = \pm\frac{\boldsymbol{L}}{2} \ln\left(\frac{1}{u}\right) \tag{5}$$

Substituting $\boldsymbol{Y} = \boldsymbol{x} - \boldsymbol{P}$ into Eq. (3), the particle location update equation is shown as

$$\boldsymbol{x}_i(t+1) = P_i(t) \pm \frac{\boldsymbol{L}_i(t)}{2} \ln\left(\frac{1}{u_i(t)}\right) \tag{6}$$

and

$$\boldsymbol{P}_i(t) = \varphi_i(t) p_i(t) + [1 - \varphi_i(t)]\boldsymbol{g}(t) \tag{7}$$

$$\boldsymbol{L}_i(t) = 2\alpha \left|c(t) - \boldsymbol{x}_i(t)\right| \tag{8}$$

$$\boldsymbol{c}(t) = \frac{1}{M}\sum_{i=1}^{M} \boldsymbol{p}_i(t) \tag{9}$$

where $i = 1, 2, \cdots, M$ is the $i$th particle, $M$ is the size of particle swarm, $t$ is the evolutionary generation. $\boldsymbol{x}_i(t)$, $\boldsymbol{P}_i(t)$ and $\boldsymbol{L}_i(t)$ are the particle current location, the local attractor location, and the characteristic length of particle aggregation state, respectively. $\boldsymbol{p}_i(t)$ is the individual best location of the particle, and $\boldsymbol{g}(t)$ is the global best location, also known as the location of guider particle. $\boldsymbol{c}(t)$ is the average best location of the entire particles at generation $t$. Both $u_i(t)$ and $\varphi_i(t)$ are the uniform random numbers in [0,1]. If $u_i(t) < 0.5$, Eq. (6) takes '+'; otherwise takes '−'. $\alpha$ is the expansion–constriction factor, which is the only parameter except the swarm size and maximum iteration. Ref. [19] proved that the global convergence can be acquired when $\alpha < 1.782$, which usually set $\alpha$ in linear decreasing at each iteration.

### 3.2. Principle of MOQPSO

As opposed to the single objective optimization converging to a single solution, the multi-objective optimization obtains a Pareto optimal set. Hence the Pareto optimal solutions in each generation should store in external file, and this file must update and maintain continually along with the particles movement so as to arrive Pareto optimal front finally. This potential Pareto front obtained by MOO algorithm is expected to be close to the real Pareto optimal front, namely, with great convergence property. Moreover, the Pareto front should be uniformly distributed in a broader range.

QPSO algorithm displays excellent global searching capability and fast convergence on single objective optimization. However, the faster convergence rate of QPSO may result in premature convergence when adopted in MOO problem. The Pareto optimal solution tends to gather in boundary solutions which reduces the population diversity. Generally, Gaussian mutation operator or chaos mechanism could be introduced to enhance population diversity and improve premature tendency. Maintaining external file via adaptive grid, clustering technique or crowded distance sorting, could prompt the distribution of Pareto solutions more