Optik 127 (2016) 4948–4953

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

A new cruise missile path tracking method based on second-order smoothing

Yang-Wang Fang[a], Dong-Dong Qiao[a], Lei Zhang[a], Peng-Fei Yang[a], Wei-Shi Peng[a,b,*]

[a] School of Aeronautics and Astronautics Engineering, Air Force Engineering University, Xi'an 710038, Shaanxi, China
[b] School of Equipment Engineering, People Armed Police Enginerring University, Xi'an 710086, Shaanxi, China

ARTICLE INFO

Article history:
Received 14 January 2016
Accepted 16 February 2016

Keywords:
Path tracking
Path smoothing
Cruise missile
Symmetrical polar polynomial curve

ABSTRACT

Path smoothing is an efficient method to improve the tracking performance of a cruise missile during the process of path planning. In this paper, a new cruise missile path tracking method based on second-order smoothing is presented. First, the path of a cruise missile smoothing method is proposed to generate flyable path with continuous curvature in terms of the symmetrical polar polynomial curve. Second, with the presented conception of virtual missile, we design an optimal controller to track the path generated from the symmetrical polar polynomial curve. Finally, numerical example is also provided to illustrate the effectiveness of the proposed method. It is shown that the proposed method is not only effective but also meeting the restriction of turning radius of the missile and precision tracking.

1. Introduction

Path planning is an efficient method for cruise missile to avoid threats and enhance its penetration probability [1]. In recent years, path planning algorithms have received a great amount of attention. According to different search methods, present path planning algorithms can be divided into two categories i.e., certainty search method and randomization search method. Certainty search method mainly includes dynamic programming and A*search method, while randomization search method consists of evolutionary algorithm, ant colony algorithm and particle swarm optimization algorithm. A*search method was proposed by Hart [2], which can decline difficulty of path searching. Dynamic programming method gained much attention in recent years due to it can obtain global optimization results. Hasircioglu [3] presented a path planning method based on Evolutionary algorithm in which B-spline was used to imitate UAV's flying path. Ant colony algorithm was proposed by Colorni [4] which used Distributed parallel computation mechanism with strong robustness; but it's easy to get local optimal results and has a long convergent time. Particle swarm optimization algorithm was first proposed by Kennedy [5] and Eberthart [6], whose model is concise and convergence speed is very quick but with low convergence precision and efficiency.

The optimal path obtained from the above algorithms is a series of waypoints in 3-D space, which are connected directly by line, so the initial path generated from these algorithms is broken line path [7]. However, cruise missile cannot follow such broken line path precisely because of the restrictions of dynamic performance and kinematics characteristics, it is essential to smooth the path. The objective of path smoothing is to generate a path whose curvature radius is greater than the minimum turning radius of the missile at every point; meanwhile, the path must meet the condition of curvature continuity, i.e. the first and second order smoothing. Since then, path smoothing method includes Dubins curve, spline curve and polynomial curve. In 1957, Dubins presented the Dubins curve [8], which can meet the restriction of maximum curvature, but its curvature is discontinuous at the connection points and it can only meet first-order smoothing, furthermore, the difficulty of path tracking was increased which leads to a big tracking error [9–11]. The path based on ordinary polynomial curve is longer than Dubins curve since it ignores the restriction of mobility performance of the missile. In contrast to methods introduced above, a method based on symmetrical polar polynomial curve is presented in this paper whose curvature is continuous and it can meet path second-order smoothing.

To overcome the above problem, new cruise missile path tracking method based on second-order smoothing is presented.

* Corresponding author.
E-mail addresses: ywfang2008@sohu.com (Y.-W. Fang), suifeng103@126.com (D.-D. Qiao), szl1985@163.com (L. Zhang), pfyang1988@126.com (P.-F. Yang), peng_weishi@163.com (W.-S. Peng).

http://dx.doi.org/10.1016/j.ijleo.2016.02.043
0030-4026/

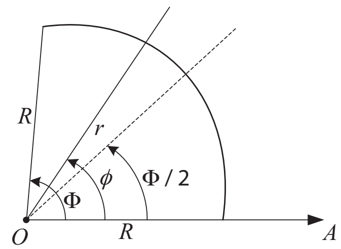

Fig. 1. Symmetrical polar polynomial curve.

This paper is organized as follows. The method of a second-order smoothing path based on symmetrical polar polynomial curve is proposed in Section 2. In Section 3, first, the linearization model of the missile is built based on the method of feedback linearization; second, a path tracking problem is transformed into a problem of tracking and controlling for a virtual missile whose concept is presented; finally an optimal controller of path tracking is designed. Simulation examples are shown in Section 4 to prove the effectiveness and periority of the proposed method. Section 5 concludes this paper.

2. Path-smoothing based on symmetrical polar polynomial curve

In this paper, a smooth path which can be tracked precisely is generated by bringing symmetrical polar polynomial curve into path smoothing process of cruise missile.

2.1. Symmetrical polar polynomial curve

A symmetrical polar polynomial curve on the plane is shown in Fig. 1, where O is a pole; OA is a polar axis; (r, ϕ) represents polar coordinates; R is the polar diameter length of the curve; and Φ is the angle between the two polar diameters. So the equation of the curve [12] in Fig. 1 is as follows

$$r(\phi) = R\left(1 + \frac{\phi^2}{2} - \frac{\phi^3}{\Phi} + \frac{\phi^4}{2\Phi^2}\right) \tag{1}$$

From formula (1) we know that $r(\phi) = r(\Phi - \phi)$, the direction angle α and the tangent curvature κ at point (r, ϕ), respectively, are as following

$$\alpha = \phi - \arctan\left(\frac{r'}{r}\right) + \frac{\pi}{2} \tag{2}$$

$$\kappa(\phi) = \frac{r^2 + 2r'^2 - rr''}{(r^2 + r'^2)^{3/2}} \tag{3}$$

the first and second order derivatives of r are as follows

$$r' = \frac{dr}{d\phi} = R\left(\phi - 3\frac{\phi^2}{\Phi} + 2\frac{\phi^3}{\Phi^2}\right) \tag{4}$$

$$r'' = \frac{d^2r}{d\phi^2} = R\left(1 - 6\frac{\phi}{\Phi} + 6\frac{\phi^2}{\Phi^2}\right) \tag{5}$$

According to formula (3), both $\kappa(0)$ and $\kappa(\Phi)$ are equal to zero by plugging $\phi = 0$ and $\phi = \Phi$ into formulas (1), (4) and (5). So the curve meets the condition of curvature continuity when it is connected with line. Meanwhile, $\kappa'(\phi)$ can be obtained by taking derivatives of formula (3)

$$\kappa'(\phi) = \frac{-r^3r' - 4rr'^3 - 3r'^3r'' + 3r^2r'r''}{(r^2 + r'^2)^{5/2}} + \frac{3rr'r''^2 - r^3r''' - rr'^2r'''}{(r^2 + r'^2)^{5/2}} \tag{6}$$

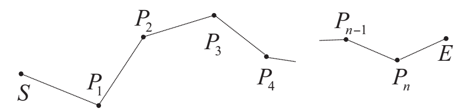

Fig. 2. Initial path.

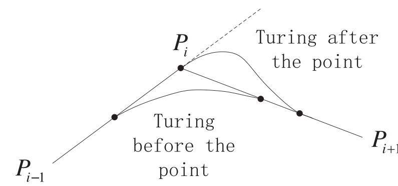

Fig. 3. Path switch principle.

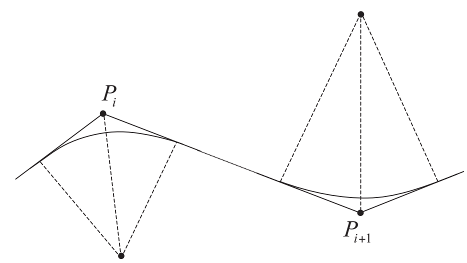

Fig. 4. Path smoothing principle.

where r''' can be obtained by taking derivatives of r''

$$r''' = \frac{d^3r}{d\phi^3} = R\left(-6\frac{1}{\Phi} + 12\frac{\phi}{\Phi^2}\right) \tag{7}$$

According to the extreme value theory and symmetrical characteristic of the curve, we know that when $\phi = \Phi/2$, the curvature κ gets the maximum value of

$$\kappa_{\max} = \kappa\left(\frac{\Phi}{2}\right) = \frac{16\Phi^2 + 384}{R(\Phi^2 + 16)} \tag{8}$$

2.2. The smoothing method of three-dimensional path

The initial path obtained from path planning algorithms is a series of path points in three-dimensional space. For the initial path $\{S, P_1, P_2, \ldots, P_n, E\}$, where S is the start point, E is the target point, $P_i(x_i, y_i)$, $i = 1, 2, \ldots, n$ is the ith path point of the cruise missile. The initial path is shown in Fig. 2.

When the missile flies along with the initial path, it needs to switch from one section to another. Path switch principles can be divided into two types, i.e. Turning after the point and Turning before the point, which is shown in Fig. 3.

Second-order smoothing method based on Turning before the points is to substitute partial sections of the initial path with polynomial curve, which can meet the condition of being tangent with the initial path. Path smoothing principle is shown in Fig. 4.

Therefore, both 2-D and 3-D path-smoothing problems can be concluded to solve the equation of polynomial curve on the plane, namely, to figure out the value of parameters R and Φ at the condition of minimum turning radius $r_{\min}$.

As is shown in Fig. 5, P_iP_{i+1}, $P_{i+1}P_{i+2}$ are sections of the initial path, the coordinates of P_i, P_{i+1} and P_{i+2} in inertial coordinates, respectively, are (x_i, y_i, z_i), $(x_{i+1}, y_{i+1}, z_{i+1})$ and $(x_{i+2}, y_{i+2}, z_{i+2})$. The plane constituted by P_iP_{i+1} and $P_{i+1}P_{i+2}$ can be defined as the moving plane, the unit normal vector $\boldsymbol{b}_p$ of the plane is

$$\boldsymbol{b}_p = \frac{\rightarrow P_{i+1}P_{i+2} \times \rightarrow P_iP_{i+1}}{\left\|\rightarrow P_{i+1}P_{i+2} \times \rightarrow P_iP_{i+1}\right\|} \tag{9}$$