Optik 127 (2016) 5070–5077

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Design and optimization of generalized prediction-based control scheme to stabilize and synchronize fractional-order hyperchaotic systems

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ARTICLE INFO

Article history:
Received 14 January 2016
Accepted 16 February 2016

Keywords:
Fractional calculus
Fractional-order chaotic systems
Prediction-based control
Multiobjective optimization

ABSTRACT

This work addresses the problem of designing a controller for the stabilization and synchronization processes of a large class of Fractional-order Chaotic (resp. hyperchaotic) Systems (FoCS). A Generalized Prediction-based Control (GPbC) law is presented in this paper. Based on Lyapunov stabilization arguments and a recent stability theorem of fractional-order systems, stability analysis of the closed-loop control system is investigated. The design and multiobjective optimization of GPbC scheme offers some superior properties such as faster finite-time convergence, higher control precision which very low energy consumption and stability conditions guarantee in control and synchronization of FoCS. Some numerical simulations are provided to confirm the validity of the proposed analytical results.

1. Introduction

Recently, fractional calculus has been rediscovered by scientists and engineers and applied in an increasing number of disciplines. Its increase use in a certain number of physical and engineering processes that are best described by fractional differential equations has motivated out its study. Modeling and control topics using the concept of Fractional-order of integral and derivative operators have been attracting more attentions. The Fractional-order Chaotic (resp. hyperchaotic) Systems (FoCS), generalization of integer-order chaotic systems, can be considered as a new alternative which significant attention has been focused on developing techniques for analysis, control and synchronization of this family of nonlinear dynamical systems. In addition, many researchers in the fractional control community have made great contributions varied from conventional, advanced to intelligent control approaches [1–9]. For example, Chen et al. in Ref. [1] investigated the chaos control of a class of Fo chaotic systems *via* sliding mode concept. Zhu et al. [2] presented an algorithm for numerical solution of fractional-order differential equation. The synchronization of fractional-order Chua oscillator is discussed. A survey of fractional dynamical systems, modeling, stability analysis and control has been presented in [3]. In [4,5], the authors investigate the stability conditions of n-dimensional fractional-order nonlinear systems with commensurate-order lying in (0, 2). The obtained results are applied to stabilizing a large class of FoCS *via* a linear state-feedback controller. In [6], the function projective synchronization between different FoCS with uncertain parameters using modified adaptive control method is studied. The adaptive function projective synchronization controller and identification parameter laws are developed on the basis of Lyapunov stability theory. A new mean-based adaptive fuzzy neural network sliding mode control is developed by Wang et al. [7], to perform the chaos synchronization process of fractional-order uncertain systems. In [8], the problems of the robust stability and stabilization of fractional order chaotic systems based on uncertain Takagi-Sugeno fuzzy model are studied. In [9], a modified sliding mode control scheme is proposed to realize complete synchronization of a class of FoCS. The prediction-based control, as an advanced technique, has been introduced by Ushio and Yamamoto [10] in order to overcome some limitations of the so called delayed feedback control derived by Pyragas [11]. Recently, a number of analytical results for the stabilization of fixed points and periodic orbits, in ordinary or fractional-order chaotic systems, using this control technique has been commonly applied due to its simplicity and efficiency [12–16]. In fact, prediction-based control approach offers some superior properties when compared to some other controllers such as faster finite-time convergence and higher control precision which very low energy consumption.

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http://dx.doi.org/10.1016/j.ijleo.2016.02.044
0030-4026/

The application of fractional-order calculus has been significantly increased and the fractional-order form of linear state-feedback control approach, or simply Fractional-order Controllers (FoC) introduced and well adressed in [17–19] is, attractively, become a major topic in control and synchronization processes of FoCS [20–24]. In particular, In [20], the problem of controlling unstable equilibrium points and periodic orbits is investigated *via* the fractional feedback of measured states. In [21], the control and synchronization of the fractional-order Lorenz chaotic system have been adressed *via* the fractional-order derivative approach. The single state FoC for chaos synchronization process based on the Lyapunov stability theory is presented by Li et al. [22]. In [23], the fractional operators are introduced to develop a general form for synchronizing a class of FoCS. The authors, also, adopt the CRONE-Oustaloup method to simulate the fractional-order systems and the fractional calculus operators. Fractional-order PI controller for locally stabilize unstable equilibrium points of a class of chaotic fractional order systems is proposed by Tavazoei and Haeri in [24].

The prediction-based control technique in its fractional-form may be seen as an alternative whose objective is to improve the performances required in control and synchronization of a large class of FoCS [15,25]. Therefore, designing a fractional-order prediction-based control method for FoCS is still, also, an open problem.

A view of the researches carried out in the field of fractional-order modeling and control, the elaboration of control law, the discretization process of fractional operators $D_t^{\pm\alpha}\bullet$, $\alpha \in \mathbb{R}$ (resp. $s^{\pm\alpha}$) and the stability analysis of fractional-order systems are the most fundamental and important issues.

Taking into consideration the previous discussion, we propose in this paper a novel method to stabilize and to synchronize a class of fractional-order chaotic and hyperchaotic systems by combining the FoC and the prediction-based control. The design and optimization of the proposed Generalized Predictive-based Controller (GPbC) is derived and based on Lyapunov stabilization arguments and genetic learning. Finally, Numerical simulations are given to show the effectiveness of our designed scheme by taking a class of fractional-order hyperchaotic systems as illustrative examples.

The rest of the paper is organized as follows. In Section 2, basic definitions and some preliminaries of fractional calculus and fractional-order systems are introduced. A description of FoCS is also included. Based on fractional approach, a new alternative of fractional model to control and synchronize a large class of commensurate and incommensurate FoCS is proposed in Section 3. The design tools of the proposed scheme are also discussed. Some numerical simulations presented to confirm the validity of the analytical results of the paper are displayed in Section 4. Finally, conclusion is given in Section 5.

2. Overview of fractional calculus and fractional-order systems

Several alternative definitions of the fractional-order integrals and derivatives exist. The three most common known definitions of fractional operators are Grünwald–Letnikov definition, Riemann–Liouville definition and Caputo definition. Next, we will recall some basic definitions, remarks and lemmas of the fractional-order calculus and FoCS systems [19,26].

Definition 1. The Riemann–Liouville (RL) fractional derivative of order $q > 0$ of a function f defined on the interval $[a, b]$ is given by

$$ {}_a^{\mathrm{RL}}D_t^q f(t) = \frac{1}{\Gamma(n-a)}\left(\frac{\mathrm{d}}{\mathrm{d}t}\right)^n \int_a^t (t-\tau)^{n-q-1}\cdot f(\tau)\,\mathrm{d}\tau \quad (1) $$

where n is the first integer larger than q, *i.e.*, $0 < (n-1) < q < n$ and $\Gamma(\bullet)$ is the Euler Gamma function.

Definition 2. The Caputo (C) fractional derivative of order $q > 0$ of a function f defined on the interval $[a, b]$ is given by

$$ {}_a^{C}D_t^q f(t) = \frac{1}{\Gamma(n-a)} \int_a^t (t-\tau)^{n-q-1}\cdot f(\tau)\,\mathrm{d}\tau \quad (2) $$

where n is the first integer larger than q.

Definition 3. The Grünwald–Letnikov (GL) approach, the most suitable method for the realization of discrete-control algorithms, is given by

$$ {}_a^{GL}D_t^q f(t) = \lim_{h\to 0} h^{-q}\cdot \sum_{i=0}^{N---} (-1)^i \cdot \binom{q}{i} \cdot f(t-i\cdot h) \quad (3) $$

where $N--- = \lceil (t-a)/h \rceil$ is the upper limit of the computational universe, $[\bullet]$ means the integer part and h is the step-time increment. $c_{i\geq 0}^q = (-1)^i \cdot \binom{q}{i}$ represents the binomial coefficients calculated according to the relation

$$ c_0^q = 1,\quad c_i^q = \left(1 - \frac{1+q}{i}\right)\cdot c_{i-1}^q,\quad \forall i > 0 \quad (4) $$

As shown by GL, RL and C definitions, the fractional-order derivatives are global operators having a memory of all past events. This property is used to model hereditary and memory effects in most materials and systems [27].

Remark 1. For the memory term expressed by a sum in (3), a 'short memory' principle introduced by Podlubny et al. [19] can be used. According to this principle, the length of system memory can be substantially reduced in the numerical algorithm to get reliable results. Some authors propose other ideas to improve the storage capacity and computation-time of fractional-order systems [28–30].

Definition 4 *([31,32])*. A direct definition of the fractional derivative $D_t^q y(t)$ is based on finite difference of an equidistant grid in $[0, 1]$. Assume that the function $y(\tau)$ satisfies some smoothness conditions in every finite interval $[0, t]$, $t \leq T$. Choosing the grid

$$ (0 = \tau_0) < \tau_1 < \cdots < (\tau_{n+1} = t = (n+1)\cdot h),\quad (\tau_{n+1} - \tau_n = h) \quad (5) $$

and using the notation of finite differences

$$ \Delta_h^q y(t) = \frac{1}{h^q}\cdot\left(y(\tau_{n+1}) - \sum_{\nu=1}^{n+1} c_\nu^q \cdot y(\tau_{n+1-\nu})\right) \quad (6) $$

where $c_\nu^q = (-1)^{\nu-1}\binom{q}{\nu}$, the GL definition reads

$$ D_R^q y(t) = \lim_{h\to 0}\frac{1}{h^q}\cdot \Delta_h^q y(t) \quad (7) $$

Lemma 1 ([31,32]. (Order of approximation)). Let the function $y(\tau)$ be smooth in $[0, T]$. Then, the GL approximation satisfies for each $0 < t < T$ and a series of step sizes h with $t/h \in \mathbb{N}$ and $t = (n+1)\cdot h$

$$ D_R^q y(t) = \frac{1}{h^q}\cdot \Delta_h^q y(t) + o(h) \quad (h \mapsto 0) \quad (8) $$

where $o(h)$ is the truncated error. If $q \in \mathbb{N}^+$, the well known finite backward differences are given. If $q = 1$, then the first-order finite difference $(y(\tau_{n+1}) - y(\tau_n))/h$ follows. If $q = 2$, then the second-order finite difference $(y(\tau_{n+1}) - 2y(\tau_n) + y(\tau_{n-1}))/h^2$, and so on.

Definition 5. The fractional-order system (resp. FoCS) of a function $x(t) \in [0, \infty)$ (system states) is defined as

$$ {}_0D_t^q x(t) = F(x(t), t),\quad x(t)\big|_{t=0} = x_0 \quad (9) $$