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Multi-sensor marginalized particle filter based on average weight optimization in correlated noise

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ABSTRACT

Particle filter is a kind of powerful and effective simulation-based method to perform optimal state estimation in nonlinear non-Gaussian state-space models. However, its main drawback is with large computational complexity and not suitable for noise correlation condition, which limits its application in the multi-sensor measurement system. Aiming at the above problem, a novel multi-sensor marginalized particle filter based on average weight optimization in correlated noise is proposed. First, marginalized particle filter is used as the basic framework of new algorithm realization by marginalizing the states appearing linearly in the dynamical system, and the objective is to reduce the calculated amount. Second, considering the rational utilization of multi-sensor measurement, the average weight optimization strategy is used to improve the adverse influence caused by random measurement noise in measuring process of particles weight. Third, combining with the model reconstruction technology, a new decoupling approach of correlated noise is designed in multi-sensor measurement. Finally, the theoretical analysis and experimental results show the feasibility and efficiency of the proposed algorithm.

1. Introduction

Nonlinear filtering problems arise in many fields including statistical signal processing, economics, statistics, and engineering such as communications, radar tracking, sonar ranging, target tracking, and satellite navigation. It can be viewed as an optimal filtering problem under a Bayesian framework, and the solving process usually need to construct the posterior probability density function (PDF) of estimated state on the basis of all the available measurement information [1,2]. In recent years, the growth of computational power has made computer intensive statistical methods feasible. Based on the sequential importance sampling technique and the recursive Bayesian filter principle, Particle filter (PF) give a usefully approximate approach to obtain the posterior PDF based on the Monte Carlo simulation. It represents the required PDF by a set of random samples (particles) with associated weights and computes estimated state based on these weighted particles [3]. Although PF is fairly easy to implement and tune, its main drawbacks are with the particle degeneration problem and the greatly computational complexity. Aiming at the particle degeneration problem, some techniques have been developed to improve the performance of particle filter. These approaches includes the optimization of proposal distribution [4,5], the construction of kernel function [6] and some intelligent optimization methods [7,8], etc. In order to reduce the calculation amount, marginalized particle filter (MPF) gives a kind of good solution by marginalize out the states appearing in the dynamical system, meanwhile, it constructs the feedback mechanism between the estimated linear vector and the nonlinear state vector [9]. Because of the construction of feedback path, the estimation of linear state vector by Kalman filter (KF) with the linear minimum-variance feature can be used to optimize the estimation of nonlinear state vector by PF. Therefore, the filter precision of system state will be promoted undoubtedly relative to the standard PF [10]. Unfortunately, the above achievements pay close attention to the single sensor measurement system for the moment, and combining with the characteristics of multi-sensor measurement system, the study of design and application for PF is relatively few.

In addition, we known that the implementation of PF need to be subject to some basic assumptions, it is important that the system noise and the measurement noise are independent identically distributed. However, the coordinate conversion between the target motion modeling and the measurement modeling in target tracking system, or the space transformation and registration of distributed measurement inevitably all lead to the correlation between the measurement noise and the system noise [11]. Aiming

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to the correlations between the system noise and the measurement noise in nonlinear system, Chen and others design a kind of new decoupling method by the rearrange the state transition equation to a new one, and removes the correlation between noise [12]. They make the decoupling method apply to the framework of PF and improve the better filtering precision.

Based on the above analysis, a novel multi-sensor marginalized particle filter based on average weight optimization in correlated noise (MMPF-AWO) is proposed. The remaining of the paper is organized as follows. First, the basic feature of MPF is briefly introduced in multi-sensor measurement in Section 1. Second, the theoretical derivation on the optimization strategy of particle weight and the decoupling method of correlated noise are, respectively, given in Section 2. In addition, the construction process of MMPF-AWO is given. Fourth, the experimental setup and simulations is presented in Section 3. The final section lists the conclusions and recommendations.

2. Marginalized particle filter in multi-sensor measurement

Consider the following nonlinear state space model with the characteristics of multi-sensor measurement.

$$\boldsymbol{x}_k^n = f\left(\boldsymbol{x}_{k-1}^n\right) + \boldsymbol{A}_{k-1}^l \boldsymbol{x}_{k-1}^l + \boldsymbol{B}_{k-1}^n \boldsymbol{u}_{k-1}^n \tag{1}$$

$$\boldsymbol{x}_k^l = \boldsymbol{A}_{k-1}^l \boldsymbol{x}_{k-1}^l + \boldsymbol{B}_{k-1}^l \boldsymbol{u}_{k-1}^l \tag{2}$$

$$\boldsymbol{z}_{k,m} = h\left(\boldsymbol{x}_k^n\right) + \boldsymbol{v}_{k,m} \quad m = 1, 2, \ldots, M \tag{3}$$

where $\boldsymbol{x}_k^n$ and $\boldsymbol{x}_k^l$ denote the nonlinear state vector and the linear state vector, respectively. $\boldsymbol{x}_k = \left[\boldsymbol{x}_k^n, \boldsymbol{x}_k^l\right]^{\mathrm{T}}$ denotes the system state vector. $\boldsymbol{f}(\cdot)$ and $\boldsymbol{A}_k^l$ denote the state-transition function of $\boldsymbol{x}_k^n$ and $\boldsymbol{x}_k^l$, respectively. $\boldsymbol{B}_k^n$ and $\boldsymbol{B}_k^l$ denote the system noise matrix of the nonlinear and linear state vector, respectively. $\boldsymbol{u}_k = \left[\boldsymbol{u}_k^n, \boldsymbol{u}_k^l\right]^{\mathrm{T}}$ denotes the system noise vector, here $\boldsymbol{u}_k^n \sim N\left(0, \sigma_{\boldsymbol{u}_k^n}^2\right)$ and $\boldsymbol{u}_k^l \sim N\left(0, \sigma_{\boldsymbol{u}_k^l}^2\right)$. $\boldsymbol{z}_k$ and $h(\cdot)$ denote the measurement vector and the measurement mapping function, respectively. $\boldsymbol{v}_{k,m} \sim N\left(0, \sigma_{\boldsymbol{v}_{k,m}}^2\right)$ denotes the measurement noise. Only some sensors with same measurement accuracy are considered in the construction of new algorithm, and suppose that $\sigma_{\boldsymbol{v}_{k,m}}^2$ is equal to $\sigma_{\boldsymbol{v}_k}^2$. Eq. (1) can be further rewritten as follows.

$$\bar{\boldsymbol{z}}_k = \boldsymbol{x}_{k-1}^n - f\left(\boldsymbol{x}_{k-1}^n\right) = \boldsymbol{A}_{k-1}^l \boldsymbol{x}_{k-1}^l + \boldsymbol{B}_{k-1}^n \boldsymbol{u}_{k-1}^n \tag{4}$$

where $\bar{\boldsymbol{z}}_k$ denotes the pseudo measurement. Therefore, combining with the above system model characteristic, KF can be applied to the linear and Gaussian system described by Eqs. (2) and (4), and PF can be applied to the linear system described by Eqs. (1) and (3). The above are the modeling and filtering mechanisms of MPF in multi-sensor measurement.

3. Multi-sensor marginalized particle filter based on average weight optimization in correlated noise

3.1. The average weight optimization strategy of particle weight

The effective sampling of particles state and the reasonable measurement of particles weight are considered as two important aspects to improve filtering precision of PF. The effective sampling key of particles state is to optimize the sampling particle by the introduction of current measurement, the proposal distribution optimization is considered as common solution. The reasonable evaluation starting point of particles weight is as far as possibly to reduce the adverse influence caused by random measurement noise on the evaluating process of particles weight [13]. In view of the characteristic of multi-sensor measurement system, it provides objectively the necessary condition to improve the influence of random measurement noise by the utilization of multi-sensor measurement. According to the realization principle of PF, meanwhile, combined with the construction of multi-senor measurement likelihood function and the average weight fusion principle, we design a new optimization method of particle weight in multi-sensor measurement. The main idea is to promote the reliability and stability of particle weight by decreasing weight variance.

First, the measurement likelihood function of sensor m is calculated by

$$p\left(\boldsymbol{z}_{k,m}|\boldsymbol{x}_{k,i}^n\right) = \exp\frac{\left(-\left(h\left(\boldsymbol{x}_k^n\right) + \boldsymbol{v}_{k,m} - h\left(\boldsymbol{x}_{k,i}^n\right)\right)^2 / 2\boldsymbol{\sigma}_{\boldsymbol{v}_k}^2\right)}{\sqrt{2\pi}\boldsymbol{\sigma}_{\boldsymbol{v}_k}} \tag{5}$$

where $\boldsymbol{x}_{k,i}^n$ denotes particles which are sampled from the nonlinear system, here $i = 1, 2, \ldots, N$ and $N \to \infty$. The weight of particle i is evaluated by the measurement of sensor m.

$$\omega_{k,i,m} = \omega_{k-1,i,m} p\left(\boldsymbol{z}_{k,m}\middle|\boldsymbol{x}_{k,i}^n\right) = \gamma_{k,i,m} \cdot \xi_{k,i,m} \tag{6}$$

where

$$\gamma_{k,i,m} = \omega_{k-1,i,m} \exp\frac{\left(-\left(h\left(\boldsymbol{x}_k^n\right) - h\left(\boldsymbol{x}_{k,i}^n\right)\right)^2 / 2\boldsymbol{\sigma}_{\boldsymbol{v}_k}^2\right)}{\sqrt{2\pi}\boldsymbol{\sigma}_{\boldsymbol{v}_k}} \tag{7}$$

$$\xi_{k,i,m} = \exp\left(-\frac{\left(2\left(h\left(\boldsymbol{x}_k^n\right) - h\left(\boldsymbol{x}_{k,i}^n\right)\right)\boldsymbol{v}_{k,m} + \boldsymbol{v}_{k,m}^2\right)}{2\boldsymbol{\sigma}_{\boldsymbol{v}_k}^2}\right) \tag{8}$$

According to the expression of $\gamma_{k,i,m}$, it is can be known as the constant when only particle i is considered. Next, combined with Eq. (5), the merged measurement likelihood function $\hat{p}\left(\boldsymbol{z}_{k,1:M}\middle|\boldsymbol{x}_{k,i}^n\right)$ is solved by

$$\hat{p}\left(\boldsymbol{z}_{k,1:M}\middle|\boldsymbol{x}_{k,i}^n\right) = \frac{\sum_{m=1}^M p\left(\boldsymbol{z}_{k,m}\middle|\boldsymbol{x}_{k,i}^n\right)}{M} \tag{9}$$

and

$$\hat{\omega}_{k,i} = \hat{\omega}_{k-1,i}\hat{p}\left(\boldsymbol{z}_{k,m}\middle|\boldsymbol{x}_{k,i}^n\right) = \frac{\gamma_{k,i,m}\sum_{m=1}^M \xi_{k,i,m}}{M} \tag{10}$$

Because the sensor accuracies are same, meanwhile the measurement noise sequences are subject to independently identically distribution (i.i.d). So the mean of $\xi_{k,i,m}$ can be supposed as $\mu_{k,\xi}$, and the mean of $\omega_{k,i,m}$ and the mean of $\hat{\omega}_{k,i}$ are written as

$$E\left[\omega_{k,i,m}\right] = \gamma_{k,i,m}\mu_{k,\xi} \tag{11}$$

$$E\left[\hat{\omega}_k^i\right] = \frac{\gamma_{k,i,m} E\left[\sum_{m=1}^M \xi_{k,i,m}\right]}{M} = \gamma_{k,i,m}\mu_{k,\xi} \tag{12}$$

$$\boldsymbol{\sigma}_{\omega_{k,i,m}}^2 = \left(\gamma_{k,i,m}\right)^2 E\left[\left(\xi_{k,i,m}\right)^2\right] - \left(\gamma_{k,i,m}\right)^2 \mu_{k,\xi}^2 \tag{13}$$

$$\sigma_{\hat{\omega}_{k,i}}^2 = E\left[\left(\hat{\omega}_{k,i}\right)^2\right] - \left(\gamma_{k,i,m}\right)^2 \mu_{k,\xi}^2 = \frac{\sigma_{\omega_{k,i,m}}^2}{M} \tag{14}$$

From the above result, we find that the variance of $\hat{\omega}_{k,i}$ has been reduced to $1/M$ of the variance of $\omega_{k,i,m}$.

3.2. The decoupling method of correlated noise

The correlation appeared on the system noise and the measurement noise can be expressed by the covariance $\boldsymbol{C}_{k,m}^n$ between them.

$$E\left[\boldsymbol{u}_k^n\left(\boldsymbol{v}_{k,m}\right)^T\right] = \boldsymbol{C}_{k,m}^n \tag{15}$$