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Transmission of an arbitrary polarized plane electromagnetic wave through absorbing non-regular one-dimensional media

A.Zh. Khachatrian [a,*], Al.G. Alexanyan [b], A.S. Avanesyan [b], N.A. Alexanyan [a,b]

[a] State Engineering University of Armenia, Teryan 105, Yerevan 0009, Armenia
[b] Institute of Physics Applied Problems of NAS of Armenia, Nersessyan 25, Yerevan 0010, Armenia

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ABSTRACT

The problem of a wave field description for a harmonic in time plane wave propagating through arbitrary one dimensional absorbing media is considered. The analysis is performed in the framework of the so-called approach of counter-propagating waves developed for a case of the normal incidence, so that the results given in this work can be considered in many respects as a generalization on this approach. The suggested approach is also applied for calculations of a space distribution of a wave field both inside and outside of different layered structures.

1. Introduction

It is well known that the description of a space distribution of a harmonic in time wave field for a one dimensional linear media can be done by means of many methods such us the classic methods for solution of ordinary differential equations, integral equations method, transfer matrix and scattering matrix methods, Green function approach, invariant imbedding method, phase function method, semi-classical method and so on [1–15]. All these methods are applied for analysis of different physical problems. Each of them has its advantages and disadvantages. Choice of this or that solution method is dictated generally by the problem statement, namely which aspect of the problem is more interesting: its calculation part, its physical aspect or the possibility to get ideas for consideration of more complex cases. Although these methods are aimed to solve the same problem existing in parallel for a long time the relation between them almost has not been discussed. More complicity this problem was explored in the resent work [16], where one the base of the so-called approach of counter-propagating waves the relation between the different methods were more transparently and completely shown.

In the present work we generalize the method developed in the papers [17,18] for the case of a harmonic in time electromagnetic wave oblique incidence on an arbitrary one dimensional medium. As it is known for that case the space shape of a wave field depends on a wave polarization. Let us consider the harmonic in time electromagnetic wave ($\vec{E}\left(\vec{r}\right)\exp\left\{-i\omega t\right\}$ is electric component, $\vec{E}\left(\vec{r}\right)\exp\left\{-i\omega t\right\}$ is magnetic component) oblique incidence on a one dimensional layer with optical properties characterizing by a space dependence and complex dielectric constant:

$$\varepsilon(x)=\begin{cases}1, & x<x_1,\\ \varepsilon'(x)+i\varepsilon''(x), & x_1<x<x_2,\\ 1, & x>x_2.\end{cases} \quad (1)$$

Supposing that the wave vector of the incident wave lies in the $(x,\ y)$ plane for a space part of a wave field outside of the layer region one can write:

$$\vec{E}\left(\vec{r}\right)=\begin{cases}\vec{E}_0\exp\left\{i\vec{k}\vec{r}\right\}+\vec{E}_r\exp\left\{i\vec{k}_r\vec{r}\right\}, & x<x_1,\\ \vec{E}_t\exp\left\{i\vec{k}_t\vec{r}\right\}, & x>x_2,\end{cases} \quad (2)$$

where $\vec{k}$, $\vec{k}_t$, $\vec{k}_r$ are the wave vectors of the incident, transmitted and reflected waves, correspondingly, which can be written as:

$$\vec{k}=\vec{k}_t=k_{0x}\vec{e}_1+k_{0y}\vec{e}_2, \quad (3)$$

$$\vec{k}_r=-k_{0x}\vec{e}_1+k_{0y}\vec{e}_2, \quad (4)$$

where

$$k_{0x}=k_0\cos\alpha,\qquad k_{0y}=k_0\sin\alpha,\quad k_0=\frac{\omega}{c} \quad (5)$$

and ω is the field frequency, c is the light velocity in free space, α is the incident angle.here the vectors $\vec{e}_1,\vec{e}_2,\vec{e}_3$ are the unit dimensionless vectors in the x, y, z directions. Note the asymptotic behavior by analogy to Eq. (2) takes place for the space component of the magnetic field $\vec{H}\left(\vec{r}\right)$ as well.

* Corresponding author. Tel.: +314 10581352..
E-mail address: ashot.khachatrian@gmail.com (A.Zh. Khachatrian).

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From the theoretical and practical points of view the more interesting for consideration are the so-called *TE* and *TM* wave fields. The main property of these waves is that they do not change polarization in the scattering process and any polarized field can be presented by means of superposition of these fields.

For the *TE* waves (which are also called the *s*-polarized waves) all the amplitudes $\vec{E}_0, \vec{E}_t, \vec{E}_r$ are perpendicular to the incident plane (x, y), i.e. they are directed along the z axis:

$$\vec{E}_0^s = E_0^s \vec{e}_3, \quad \vec{E}_t^s = E_t^s \vec{e}_3, \quad \vec{E}_r^s = E_r^s \vec{e}_3, \tag{6}$$

where E_0^s, E_t^s, E_r^s are the scalar quantities, which define the transmission and reflection amplitudes for the *s* wave:

$$t^s = \frac{E_t^s}{E_0^s}, \quad r^s = \frac{E_r^s}{E_0^s}. \tag{7}$$

For the *TM* waves (which are also called the *p*-polarized waves) all the amplitudes of the field magnetic components $\vec{H}_0, \vec{H}_t, \vec{H}_r$ are perpendicular to the incident plane (*x*,*y*), which is same that they are parallel to *z* axis;

$$\vec{H}_0^s = H_0^s \vec{e}_3, \; \vec{H}_t^s = H_t^s \vec{e}_3, \; \vec{H}_r^s = H_r^s \vec{e}_3, \tag{8}$$

where H_0^s, H_t^s, H_r^s define the amplitudes of the incident, transmitted and reflected waves, so that for the transmission and reflection amplitudes one can write:

$$t^p = \frac{H_t^p}{H_0^p}, \quad r^p = \frac{H_r^p}{H_0^p}. \tag{9}$$

It is well known that when the optical properties of the layer depend on one coordinate only (see Eq. (1)), then the space dependence of the wave field with asymptotic behavior as in Eq. (2) is defined by wave equations containing one space coordinate. In the regions outside of the layer volume the field has components in the form of a plane wave (see Eq. (2)), so that the electric and magnetic components of *s* and *p* wave fields are written as:

$$\vec{E}^s\left(\vec{r}\right) = \exp\left\{ik_{0y}y\right\}\, E^s(x)\vec{e}_3, \tag{10}$$

$$\vec{H}^p\left(\vec{r}\right) = \exp\left\{ik_{0y}y\right\}\, H^p(x)\vec{e}_3. \tag{11}$$

By using the Maxwell equations one can write the following wave equation (see, for example, [18]) for the *s*-polarized field:

$$\frac{d^2E^s(x)}{dx^2} + \left(k_{0x}^2 - U(x)\right)E^s(x) = 0 \tag{12}$$

where

$$U(x) = \frac{\omega^2\left(1-\varepsilon(x)\right)}{c^2}. \tag{13}$$

For the *p*-polarized field the Maxwell equations lead to the following wave equation:

$$\frac{d}{dx}\left(\frac{1}{\varepsilon(x)}\frac{dH^p(x)}{dx}\right) + \left(k_{0x}^2 - V(x)\right)H^p(x) = 0, \tag{14}$$

where

$$V(x) = \frac{\omega^2}{c^2}\frac{1-\varepsilon(x)}{\varepsilon(x)}\sin^2\alpha. \tag{15}$$

Note that in Eqs. (12) and (14) k_{0x} is the x component of the wave vector $\vec{k}$: $k_{0x}^2 = \omega^2\cos^2\alpha/c^2$. As it follows from Eq. (1), when $x < x_1$ and $x < x_2$ the quantity $V(x)$ in Eq. (15) does not depend on incident angle and $V(x) = 0$.

The standard conditions imposing on the solutions of Eqs. (12) and (14) have the following form:

$$E^s(x+0) = E^s(x-0), \quad \left.\frac{dE^s(x)}{dx}\right|_{x+0} = \left.\frac{dE^s(x)}{dx}\right|_{x-0}, \tag{16}$$

$$H^p(x+0) = H^p(x-0), \quad \left.\frac{1}{\varepsilon(x+0)}\frac{dH^p}{dx}\right|_{x+0} = \left.\frac{1}{\varepsilon(x-0)}\frac{dH^p}{dx}\right|_{x-0}. \tag{17}$$

These equalities should be performed for any value of x even for the values when the medium dielectric constant changes discontinuously.

In this work we investigate Eq. (14) by the method developed in the works [17] for solving the one-dimensional Schrodinger equation (see Eq. (15)). For clarity below in the first paragraph we will introduce similar results obtained by the method of counter-propagating waves, where a linear set of differential equations for the amplitudes of the counter-propagating waves are derived.

2. Brief presentation of the method of counter-propagating waves

The main idea of this method is a change of the wave equation on a set of two differential equations, so that for any space point the unknowns would be the amplitudes of the counter-propagating waves. Let us consider the couple of functions $c(x)$ and $d(x)$, which satisfy the following set of linear differential equations:

$$\frac{dc(x)}{dx} = -\frac{iU(x)}{k_{0x}}c(x) - \frac{iU(x)}{k_{0x}}d(x)\exp\left\{-i2k_{0x}x\right\}, \tag{18}$$

$$\frac{d\,d(x)}{dx} = \frac{iU(x)}{k_{0x}}d(x) + \frac{iU(x)}{k_{0x}}c(x)\exp\left\{i2k_{0x}x\right\}. \tag{19}$$

As it was mentioned the method of counter-propagating waves has been developed for consideration of the *s* wave Eq. (12). It is easy to check that the function $E^s(x)$, constructed with help of the functions $c(x)$ and $d(x)$ in accordance with the following formula:

$$E^s(x) = c(x)\exp\left\{ik_{0x}x\right\} + d(x)\exp\left\{-ik_{0x}x\right\}, \tag{20}$$

satisfies Eq. (12).

It is important to note that analysis of the wave Eq. (12) on the base of the set of Eqs. (18) and (19) is mainly motivated by the fact that the derivation of the wave function written by means of the functions $c(x)$ and $d(x)$ has the form of

$$\frac{dE^s(x)}{dx} = ik_{0x}\left(c(x)\exp\left\{ik_{0x}x\right\} - d(x)\exp\left\{-ik_{0x}x\right\}\right). \tag{21}$$

The set of Eqs. (18) and (19) can be solved with different initial or boundary conditions, which define problem statement of the asymptotic behavior of the wave function. In a physical point of view the more interesting is a boundary type of problem when the values of the functions $c(x)$ and $d(x)$ are given in the different regions on the left and right of the layer:

$$c(x_1) = c_0, \quad d(x_2) = d_0. \tag{22}$$

Note that the quantities c_0 and d_0 correspond to the magnitudes of the waves converging to the layer. The wave field determined in accordance with condition (22) is called as a converging solution. It is easy o check that in this case the space dependence of the wave field satisfies the following integral equations

$$E^s(x) = c_0\exp\left\{ik_{0x}x\right\} + d_0\exp\left\{-ik_{0x}x\right\} + \int_{x_1}^{x_2} V\left(x'\right)G_0^{(+)}\left(x,x'\right)E^s\left(x'\right)dx',$$