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Propagation of a fundamental laser mode and its numerical simulation by the angular spectrum technique

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ABSTRACT

This research aims to make a comparison between the analytical and numerical methods of simulation of the Gaussian beam propagation in free space. This comparison brings evaluation of the accuracy and applicability limitations of the angular spectrum diffraction algorithm. In this approach, planar harmonic waves are assumed as basic components of a propagating wave, so the algorithm combines the angular spectrum technique and the 2D fast Fourier transformation procedures. It means a quite different approach than the well-established Huygens–Fresnel–Rayleigh–Sommerfeld one, based on spherical waves. The Gaussian beam is a rare case of the exact analytical 3D solution of a specific wave diffraction problem, in paraxial scalar approximation. Gaussian beam form has been chosen here as an evaluation reference. It allows to avoid comparisons between two numerical procedures, both of which may suffer from imperfections. In this comparison, the respective 2D cross-sections of calculated 3D complex valued wave fields distributions are presented. Also values of specific Gaussian beam parameters like full phase and Gouy phase shift and wave-front curvature radius are presented. The comparison between that form and the results of the angular spectrum algorithm brings information about the performance quality of that algorithm.

1. Introduction

The advent of the laser in the early sixties of the 20th century has led to applications of that new kind of light source in multiple fields of science and engineering [1]. Basic laser mode form, denoted as TEM_{00} and its mathematical representation known as the Gaussian beam is commonly used in theoretical and experimental optics by many researchers [2–5]. Such beam is described through a unit electric field vector multiplied by a scalar, in general complex, function that satisfies the paraxial scalar wave equation [6–8]. That distribution is obtained as the basic eigenmode solution of the axially symmetric laser cavity [9]. In the optical field theory development there exist several ways of diffraction field calculations based on the Huygens principle, with the spherical wave playing the role of elementary solution of the wave equation. Those are known as Fresnel diffraction (near-field) [10], Fraunhofer diffraction (far-field) [11] with several modifications introduced by Rayleigh, Sommerfeld [12] and among Polish authors Rubinowicz [13]. In contrast, the angular spectrum (AP) approach is based on decomposition of the optical field in to planar waves [14]. Diffraction integrals, that were earlier solved analytically only for some special cases, are then calculated numerically but still with high computing costs. The fast Fourier transform (FFT) algorithm rediscovered by Cooley and Tukey [15] in 1965 dramatically reduced numerical operation counts of the discrete Fourier transformation (DFT). The Fresnel diffraction integral can be viewed as 2D convolution of the initial optical field distribution with the propagation function, and may be effectively calculated with the application of the Fourier-convolution theorem [16]. The AS technique makes use of the Fourier decomposition of the initial optical field to elementary wave equation solutions in the form of planar waves, and then after propagation, their Fourier superposition [17].

In this paper we focus on the numerical modeling of Gaussian beam propagation between the initial plane at $z=0$ and the parallel observation plane at z. The AS results are compared with the closed form of the Gaussian beam obtained theoretically from the paraxial scalar Helmholtz wave equation [18]. Discussed here AS algorithm allows for calculation of the optical field diffraction at the short distance, starting from just behind the object [19–23]. 2D FFT algorithm is applied here first to achieve the AS representation of the initial field distribution and then the inverse 2D FT, to obtain superposition of all propagated planar wave components in the observation plane. Both of those transformations are

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performed by use of the application of 2D FFT algorithm, so the whole numerical procedure will be called here the AS FFT algorithm. The application of two 2D FFTs in that algorithm highly reduces the computation time, but one has to keep in mind that on the other hand the discrete nature of the FFT and the limited domain of the FFT algorithm may influence the accuracy of the AS FFT results and limit applicability of that algorithm. Such limitations are studied here through the comparison of the AS FFT algorithm results of Gaussian beams propagation with respective closed form solutions of the paraxial scalar Helmholtz wave equation.

2. Angular spectrum algorithm

2.1. Planar wave spectrum propagation

Let us assume that the light propagates in free space between two planes: the first plane which is located at $z=0$ to the second plane parallel to the first one at some other $z>0$. Such wave can be decomposed in 3D to monochromatic plane wave components [24]

$$E(x,y,z)=E_0\exp\left[i2\pi\left(\nu_x x+\nu_y y+\nu_z z\right)\right], \tag{1}$$

where the spatial frequency vector $\boldsymbol{\nu}$ is related to the wave vector $\mathbf{k}$

$$\boldsymbol{\nu}=\left(\nu_x,\nu_y,\nu_z\right)=\frac{1}{2\pi}\left(k_x,k_y,k_z\right). \tag{2}$$

Let the initial optical field distribution be described through 2D complex amplitude $E_0(x,y;z=0)$. It can be decomposed into spatial harmonics by 2D FT

$$\tilde{E}_0\left(\nu_x,\nu_y;z=0\right)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}E_0\left(x,y;z=0\right)\exp\left[-2\pi i\left(\nu_x x+\nu_y y\right)\right]dxdy. \tag{3}$$

This set of harmonics is usually called "the angular spectrum" of the initial field [25]. In propagating wave, every harmonics evolves with the distance

$$\tilde{E}_0\left(\nu_x,\nu_y;z\right)=\tilde{E}_0\left(\nu_x,\nu_y;z=0\right)\exp[i2\pi(\nu_z z)]. \tag{4}$$

For a monochromatic wave of the wavelength λ,

$$\tilde{E}_z\left(\nu_x,\nu_y;z\right)=\tilde{E}_0\left(\nu_x,\nu_y;z=0\right)\exp\left[i2\pi z\sqrt{\frac{1}{\lambda^2}-\nu_x^2-\nu_y^2}\right] \tag{5}$$

The waves superposition at the distance $z>0$ can be obtained by the inverse 2D FT

$$E_z(x,y;z)=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\tilde{E}_0\left(\nu_x,\nu_y;z=0\right)\exp\left[i2\pi z\sqrt{\frac{1}{\lambda^2}-\nu_x^2-\nu_y^2}\right]\times\exp\left[2\pi i\left(\nu_x x+\nu_y y\right)\right]\right\}d\nu_x d\nu_y \tag{6}$$

In the case of high spatial frequencies, the value under the root square becomes negative and the component related to them is an evanescent wave that decays fast with the distance z. So for the propagation distance z greater than few wavelengths, it is enough to consider domain $\nu_x^2+\nu_y^2<\frac{1}{\lambda^2}$. In the case when the object spatial frequencies are confined to $\nu_x^2+\nu_y^2\ll\frac{1}{\lambda^2}$,, the paraxial version of the AS algorithm may be applied through the substitution

$$\sqrt{\frac{1}{\lambda^2}-(\lambda\nu_x)^2-\left(\lambda\nu_y\right)^2}\approx\frac{1}{\lambda}-\frac{\lambda\nu_x^2}{2}-\frac{\lambda\nu_y^2}{2} \tag{7}$$

This approximation considerably simplifies the analytical integration of the AS Fourier integral, which gets the form

$$E_z(x,y;z)=\exp\left(\frac{i2\pi z}{\lambda}\right)\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left\{\tilde{E}_0\left(\nu_x,\nu_y;z=0\right)\exp\left[-i\pi\lambda z\left(\nu_x^2+\nu_y^2\right)\right]\times\exp\left[2\pi i\left(\nu_x x+\nu_y y\right)\right]\right\}d\nu_x d\nu_y. \tag{8}$$

2.2. Numerical AS algorithm performance conditions

The whole procedure represented by Eq. (6) involves two consecutive 2D FTs: first forward and after propagation inverse. The domain of integration in the first FT may be limited to the area $|x|<L_x$ and $|y|<L_y$ related to the initial field support. It has to be also assumed that $E_0(x,y;z=0)$ is band limited in the (ν_x,ν_y) with $2B_x$ and $2B_y$ representing the widths respectively in the ν_x and ν_y directions. According to the Whittaker–Shannon theorem, the band limited function may be fully represented in discrete representation if its sampling frequency is higher than $2B_x$ and $2B_x$ in respective directions. Then the total number of significant samples required to represent $E_0(x,y;z=0)$ is [17]:

$$M=16L_xL_yB_xB_y \tag{9}$$

Assuming $L_x=L_y=L$ and $B_x=B_y=B$ full representation of $E_0(x,y;z=0)$ requires a complex valued array of $N\times N$, with $N=LB$. For example, to represent numerically a typical experimental initial distribution, with $L=1$ cm and $B=1/\mu$m, one would need the array of about 10 000 by 10 000 samples. That means 10^8 complex values or about 1GB of memory in the case of the float precision representation.

By the fulfillment of limited support and bandwidth conditions, both FTs in Eq. (6) can be performed numerically with the application of the 2D FFT algorithm [15]. The quality of such procedure will be checked further in reference to the finite form of a well-known Gaussian mode in 3D.

2.3. Gaussian beam form

There are several parameters related to the Gaussian beam distribution, but only two of them are considered as independent. Those are the size of the beam waist W_0 and the light wavelength λ.

The beam waist radius W_0 defines the size of the beam at its symmetry plane $z=0$, where complex amplitude is given in the form

$$E_G(x,y,z=0)=E_0\exp\left[-\frac{x^2+y^2}{W_0^2}\right], \tag{10}$$

where, W_0 is the beam waist radius, and E_0 is the peak electric field amplitude at the axis.

Light field of the wavelength λ, or wave number $k=2\pi/\lambda$, that at $z=0$ is described by Eq. (10), propagates into the third dimension and the solution of the diffraction problem in this case gives finite form of the complex amplitude distribution known as the Gaussian beam [26]:

$$E_G(x,y,z)=E_0\frac{W_0}{W(z)}\exp\left[-\frac{x^2+y^2}{W^2(z)}\right]\exp\left[-ik\frac{x^2+y^2}{2R(z)}\right]\exp\left\{-i\left[kz-\varphi_G(z)\right]\right\}, \tag{11}$$