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Short Note

# Backstepping sliding mode lane keeping control of lateral position error with dynamic of tire steering device

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ABSTRACT

A kind of backstepping sliding mode control law was proposed to solve the lane keeping problem where the automatic driving system was simplified to be a kind of linear lateral dynamic model. And the dynamic response characteristic of tire steering system was described to be a first order system with a proper delay time constant. First, the complex system was divided into a relative accurate subsystem and an uncertain subsystem. Second, the proposed backstepping sliding mode method was proposed to eliminate the lateral position error which can make use of the advantage of sliding mode method that it can improve the robustness of the whole system. The backstepping strategy was adopted for the accurate inertial delay dynamic of tire steering system. Third, a kind of soft function was used to reduce the oscillation caused by sliding mode control. And a Lyapunov function was constructed to guarantee all signals of close loop system are stable. At last, 100 times of random numerical simulations were done to testify the rightness and stability of the proposed method applied in the situation that the tire coefficients are uncertain.



## 1. Introduction

In recent years, the scholars at home and abroad have been attracted by the related technology of automatic driving for vehicles, which the lane keeping control is one of the key technologies of automatic driving for vehicles. By the offset between the actual direction of vehicle moving and the central direction of the lane, this technology calculates the corner of the steering wheel with corresponding control algorithm, then based on which the vehicle is controlled to drive safely following the lane. In the literatures [1–7], for lateral dynamics of vehicles with two-degrees-of freedom front wheels steering, it was considerated the perturbation of parameters in the model of vehicle produced from the external disturbance in the moving, then the lateral position deviation controller was designed to realize the lateral safe control with lane keeping of vehicles, using adaptive, fuzzy control, neural network and sliding mode control methods respectively.

On that basis, in the literature [8], it was considerated the dynamic characteristic of steering in the vehicle furthermore. The first-order inertial component with its time constant of 0.1 s was chosed to simulate the physical delayed characteristic of the steering, and the bipolar sigmoid function was adopted to instead the sign function in the conventional sliding mode control, then the sliding mode controller with self-correction was designed to improve the buffeting phenomenon in the output of the sliding mode controller. It was described intensively the effect, which was produced by the self-correction law with switching gain and boundary layer thickness, that eliminating the buffeting and saturation reducing the working load. However, it was not analysed in-depth the negative effect by introducing the dynamic characteristic of the steering, neither was how to take the corresponding measure for dealing with the dynamic delay of the steering in the design of control law.

The introducing of the dynamic delay of the steering, greatly increasing the difficulty of system design and stability assurance. The reasons were as below, first the order of the system was increased, which leading to increase the complexity of the design and analysis. Secondly, in the previous literatures, while using sliding mode control, the lateral position error with its derivative were composed to construct the sliding mode surface, that is, corresponding the position and the velocity, which possessing the obvious physical significance. While newly adding the dynamic characteristic of the steering to become the third order, but whether introducing the rotational angle or the rotational palstance to the sliding mode surface, both would be difficult to fit the position and the velocity of the existing vehicle, which making the design

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complex. The reason of fitting difficultly was mainly because that the position and the velocity describing the information of vehicle, while the rotational angle and the rotational palstance showing the information of the physical characteristic of the steering, the latter were hard to form the differential relations. Thus, they were difficult to compose the sliding mode surface. Thirdly, the second derivative of the lateral position error could be introduced into the sliding mode surface that is adding the acceleration information. But it meant that the acceleration sensor should be added, which would not only increase the complexity of the algorithm, but also raise the cost of the control.

Based on the above analysis, a kind of backstepping sliding mode compound control was proposed in this paper, which retained the original form of the sliding mode surface. In this method, sliding mode control was used to reduce the order of the mode of dynamics for vehicle movement to the first order. This reduced first-order mode, with the introduced first-order inertial characteristic of the steering, was formed as a second-order system. Then the backstepping control was adopted to working backward progressively, and the control law was designed for the new system according to the Lyapunov theory, which realized the safe control of the lateral position error of the lane. Finally, using the technology of the computer simulation, the analysis with Digital Simulation was performed to verify the correctness and effectiveness of the method proposed in this paper. The stiffness coefficient of the tire defined or random, both were considerated in the simulation.

## 2. Model description

Without considering the declining of the road, the lateral position error and the lateral yaw angle error were chosen as the basic state variables to describe the model of lateral motion for vehicle. The mode was written as the following [9] formula:

$$\begin{bmatrix} \dot{\psi}_r \\ \ddot{\psi}_r \\ \dot{y}_s \\ \ddot{y}_s \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ \frac{2a_2}{I_z} & -\frac{2a_3}{I_z v_x} & 0 & -\frac{2a_2}{I_z v_x} \\ 0 & 0 & 0 & 1 \\ \frac{2a_1}{m} & -\frac{2a_2}{m v_x} & 0 & -\frac{2a_1}{m v_x} \end{bmatrix} \begin{bmatrix} \psi_r \\ \dot{\psi}_r \\ y_s \\ \dot{y}_s \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{2C_f l_f}{I_z} \\ 0 \\ \frac{2C_f}{m} \end{bmatrix} \delta + \begin{bmatrix} 0 \\ -\frac{2a_3}{I_z v_x} \\ 0 \\ -\left(v_x + \frac{2a_2}{m v_x}\right) \end{bmatrix} \dot{\psi}_d + \begin{bmatrix} 0 \\ -1 \\ 0 \\ 0 \end{bmatrix} \ddot{\psi}_d \tag{1}$$

In which, $\psi_r$ was shown as the lateral yaw angle error; $y_s$ was shown as the lateral position error; $I_z$ was described as the moment of inertia for vehicle; $v_x$ was indicated as the longitudinal velocity for vehicle; $m$ was meant for the mass of the vehicle; $l_f$ and $l_r$ were shown as the distance from the center of mass to the front axle and the back axle respectively in vehicle; $C_f$ and $C_r$ were written as the stiffness of the front wheel and the back wheel; $\delta$ was shown as the rotating angle of both the front wheel and the back wheel; $\psi_d$ was the desired yaw angle, $\chi$ was the curvature of the center line in the lane, then the following formulas could be obtained as $\dot{\psi}_d = v_x \chi$, $a_1 = C_f + C_r$, $a_2 = C_f l_f - C_r l_r$, $a_3 = C_f l_f^2 + C_r l_r^2$.

Referenced from the literature [8], it was considerated further the characteristic of inertial lag from the steering to the front wheel angle $\delta_z$. The first-order inertial component was introduced to describe its dynamic characteristic shown as below:

$$\dot{\delta} = \frac{1}{T_e} u - \frac{1}{T_e} \delta$$

In which, $u$ was the input of the system. For easily to be understood, yaw error velocity and lateral position error velocity were defined as below:

$$\dot{\psi}_r = \omega, \quad \dot{y}_s = v_y \tag{2}$$

Then the model shown above could be rewritten as the following mode:

$$\dot{\omega} = \frac{2a_2}{I_z} \psi_r - \frac{2a_3}{I_z v_x} \omega + \frac{2a_2}{I_z v_x} v_y + \frac{2C_f l_f}{I_z} \delta_z - \frac{2a_3}{I_z v_x} \dot{\psi}_d - \ddot{\psi}_d \tag{3}$$

$$v_y = \frac{2a_1}{m} \psi_r - \frac{2a_2}{m v_x} \omega - \frac{2a_1}{m v_x} v_y + \frac{2C_f}{m} \delta_z - \left(v_x + \frac{2a_2}{m v_x}\right) \dot{\psi}_d \tag{4}$$

$$\dot{\delta} = \frac{1}{T_e} u - \frac{1}{T_e} \delta \tag{5}$$

The task of the control with the error from the lateral position in the lane was to design the law $u = f(x)$, in which $x$ was all or part of the states be measured, which made the lateral position error $y_s$ converged to 0 under arbitrary initial position.

In view of the physical characteristic in the control system of lane keeping, the above mode (2–5) could be divided into two parts. The first part was (2–4), it contained a large number of dynamic coefficients in vehicle, and part of the coefficients was hard to identify because of the influence from the road friction, crosswind, etc. Thus the first part was defined as the uncertain subsystem. The second part was the model (5), it only included the inertial time constant $T_e$, which was related with the reaction rate of the steering in front wheel, and which was relatively fixed after production. So the second part was defined as the relatively determinated subsystem. For these two subsystems, sliding mode control and backstepping control were adopted separately, and they were mixed together as one system by choosing Lyapunov function.

## 3. The design of the control law for backstepping sliding mode

Without loss of generality, the expected value of the lateral position error $y_s$ was defined as $y_{rin}$. And without special introductions, $y_{rin} = 0$. The error variable was defined as $e_y = y_s - y_{rin}$, then the following equation could be obtained.

$$\dot{e}_y = \dot{y}_s - \dot{y}_{rin} \tag{6}$$

The sliding surface was chosen as below

$$s = Ce_y + \dot{e}_y \tag{7}$$

In which, $C$ was the parameter for the sliding surface. The derivation of the equation (7) was written as

$$\dot{s} = C\dot{e}_y + \ddot{e}_y \tag{8}$$

Eq. (6) was substituted to Eq. (8), then we could get the following formula.

$$\dot{s} = Cv_y - C\dot{y}_{rin} + \dot{v}_y - \ddot{y}_{rin} \tag{9}$$

Thus the following equation was obtained.

$$\dot{s} = Cv_y - C\dot{y}_{rin} - \ddot{y}_{rin} + \frac{2a_1}{m} \psi_r - \frac{2a_2}{m v_x} \omega - \frac{2a_1}{m v_x} v_y + \frac{2C_f}{m} \delta_z - \left(v_x + \frac{2a_2}{m v_x}\right) \dot{\psi}_d \tag{10}$$