Optik 127 (2016) 2448–2452

Contents lists available at ScienceDirect

Optik

journal homepage: www.elsevier.de/ijleo

Environment generated quantum correlations in bipartite qubit-qutrit systems

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ARTICLE INFO

Article history:
Received 21 May 2015
Accepted 17 November 2015

Keywords:
Entanglement
Quantum discord
Decoherence

ABSTRACT

The dynamics of entanglement and quantum discord for qubit-qutrit systems are studied in the presence of phase damping and amplitude damping noises. Both one way and two couplings of the marginal systems with the environments are considered. Entanglement sudden death is unavoidable under any setup, however, the required time span depends on the way of coupling. On the other hand, the dynamics of quantum discord strongly depends both on the nature of environment and on the number of dimensions of the Hilbert space of the coupled marginal system. We show that freezing and invariance of quantum discord, as previously reported in the literature, are limited to some special cases. Most importantly, it is noted that under some particular coupling the existence of environment can guarantee the generation of nonclassical correlations.

1. Introduction

Quantum mechanics is one of the theories that is more perplexing and yet very fascinating which provides a complete physical description to fundamental phenomenon taking place at the atomic levels. Indeterminism and nonlocality are the two fundamental and controversial concepts of quantum theory. In quantum theory the sources of nonlocality are Aharonov-Bohm effect and entanglement. Entanglement has been recognized as the first candidate of non-classical correlations and is considered as a vital resource for quantum information science [1]. It has been widely investigated and a number of schemes have been presented for its detection, quantification and applications [2–4].

A generic quantum state ρ is an object which is characterized not only by quantum but also by classical correlations. From quantum information point of view, distinguishing these two types of correlations is of utmost importance. One way to do this is to use entanglement versus separability formalism introduced by Werner [5]. Moreover, in a quantum state there exist correlations that are not captured by the measures of entanglement and are still useful in carrying out certain quantum information tasks. Quantum discord, with no classical analog, introduced by Ollivier and Zurek [6] and, independently, by Henderson and Vedral [7] for bipartite systems is a more general measure of quantum correlations which also captures those that do not come in the domain of quantum entanglement. It has been used as a resource for certain quantum computation models [8], encoding of information onto a quantum state [9] and quantum state merging [10,11]. Being an important measure of quantum correlations, the domain of the measurement of quantum discord has been recently extended to continuous variable systems of Gaussian and non-Gaussian states [12–14]. The study of the behavior of quantum discord for a two mode squeezed state in noninertial frames shows that quantum discord asymptotes to zero in the limit of infinite acceleration [15]. A detail of other studies related to quantum discord, alternative measures of quantum correlations and its behavior in different setups are discussed in Refs. [17–27].

The irreversible loss of fundamental quantum features such as quantum superposition by a quantum system when it interacts with an environment is one of the big issues in the practical implementation of different protocols based on these quantum features. The utility of entanglement in different quantum information tasks as a resource decreases when the system interacts for considerably long time with its environment. The effect of environment on entanglement between the components of a composite system leads to a number of undesirable consequences, such as entanglement loss, entanglement sudden death and even the rebirth of entanglement [28–31]. The study of the dynamics of quantum entanglement and quantum discord in Markovian environment shows that, unlike entanglement, quantum discord is immune to sudden death [32]. A sudden transition between classical and quantum loss of correlations for a class of Bell diagonal

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http://dx.doi.org/10.1016/j.ijleo.2015.11.178
0030-4026/

states under local dephasing noise has also been reported [33]. This behavior predicts that there exists a finite time interval, in which only classical correlations decay and quantum discord is frozen despite the presence of a noisy environment. In Ref. [34] the authors have shown that depending on the initial state, the quantum discord, in the presence of non-Markovian purely dephasing environment, might get frozen forever at a positive value. The behavior of quantum discord and entanglement in the presence of local qutrit dephasing noise is studied in Refs. [35,36]. It is shown that the entanglement for the system vanishes in a finite time interval but the quantum discord remain invariant throughout the time evolution.

In this paper, we study the behaviors of entanglement and quantum discord for a particular bipartite state, which consists of a qubit and a qutrit as marginal systems where each interacts with a local environment. The influences of two types of noisy environments, the dephasing and the amplitude damping noises are investigated. Both one way and two ways coupling of the marginal systems in the form of local and multilocal environments are considered. We find that the freezing and decay, as reported previously, in the presence of a noisy environment are not the only aspects of quantum discord. It is found that the behavior of quantum discord strongly depends on the dimensions of the Hilbert space of the marginal system. We show that maneuvering the type of coupling between the system and a particular environment can lead to the generation or destruction of quantum discord.

2. Measures of quantum correlations

In this section we briefly review the quantifiers for entanglement and nonclassical correlations. Many entanglement measures for quantifying entanglement of bipartite states exist in the literature. However, we will use negativity which is a reliable measure of entanglement of bipartite states of any dimensions, provided that the state has a negative partial transpose. The partial transpose of a bipartite density matrix $\rho_{m\nu,n\mu}$ over the second qubit B is given by $\rho^{T_B}_{m\mu,n\nu} = \rho_{m\nu,n\mu}$ and for the first qubit, it can similarly be defined. For a bipartite state ρ^{AB}, the negativity $\mathcal{N}(\rho^{AB})$ is defined as twice the absolute sum of the negative eigenvalues of partial transpose of ρ^{AB} with respect to the smaller dimensional system,

$$\mathcal{N}(\rho^{AB}) = \sum_i |\lambda_i| - \lambda_i, \tag{1}$$

where λ_i are the eigenvalues of the partial transposed density matrix.

The nonclassical correlation are quantified by discord. The discord $\mathcal{D}(\rho^{AB})$ for a bipartite state ρ^{AB} is defined as the difference between total correlations $I(\rho^{AB})$ and the classical correlation $C(\rho^{AB})$,

$$\mathcal{D}(\rho^{AB}) = I(\rho^{AB}) - C(\rho^{AB}). \tag{2}$$

The quantum mutual information $I(\rho^{AB})$ is a measure of total amount of classical and quantum correlations in a quantum state. Mathematically, it is given by

$$I(\rho^{AB}) = S(\rho^{A}) + S(\rho^{B}) - S(\rho^{AB}), \tag{3}$$

where $S(\rho) = -Tr(\rho \log_2 \rho)$ is the von Neumann entropy of the system in the state ρ and $\rho^{A(B)} = Tr_{B(A)}(\rho^{AB})$ are the two marginal states of the composite system. The classical correlation by definition is the maximal information that one can obtain and is mathematically given by [6,7]

$$C_B(\rho^{AB}) = S(\rho^{B}) - \min_{\{\Pi^A_k\}} \sum_k p_k S(\rho^B_k), \tag{4}$$

where $\rho^B_k = Tr_A((\Pi^A_k \otimes I^B)\rho^{AB}(\Pi^A_k \otimes I^B))/p_k$ is the postmeasurement state of subsystem B after obtaining the outcome k on subsystem A with probability $p_k = Tr((\Pi^A_k \otimes I^B)\rho^{AB}(\Pi^A_k \otimes I^B))$. The set $\{\Pi^A_k\}$ are projectors onto the space of marginal state A and I^B is the identity operator for the space of marginal state B of the composite system. It is important to mention that for a general mixed state, the measure of classical correlations is not symmetric, that is, $C_A(\rho^{AB}) \neq C_B(\rho^{AB})$, as a result, the quantum discord depends on which marginal state the projective measurement is carried on. However, it is known that $\mathcal{D}_A(\rho^{AB}), \mathcal{D}_B(\rho^{AB}) \geq 0$ and $\mathcal{D}_A(\rho^{AB}) = \mathcal{D}_B(\rho^{AB}) = 0$ if and only if ρ^{AB} is a classical-quantum state. We will investigate the quantity $C_B(\rho^{AB})$ for qubit-qutrit states such that the projective measurement is made on the qubit marginal state of the composite system. The measurement operators Π_k ($k = 1, 2$) in the qubit space can be expressed as follows

$$\Pi_k = \frac{1}{2}(I \pm \sum_j n_j \sigma_j), \tag{5}$$

where the $\pm$ sign corresponds to $k = 1, 2$, respectively and σ_j ($j = 1, 2, 3$) are the three Pauli spin matrices. The vector n defines a unit vector on Bloch sphere having components $n = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)^T$ with $\theta \in [0, \pi]$ and $\phi \in [0, 2\pi]$. The main obstacle in obtaining quantum discord for a general quantum state lies in the minimization procedure, which is taken over all possible von Neumann measurements, of the quantum conditional entropy of Eq. (4). The analytical expressions for classical correlation and quantum discord are only available for two-qubit Bell diagonal state and a seven-parameter family of two-qubit X states [16,19] till now. For the simple qubit-qutrit states that we consider in our work, we will calculate the quantum discord via numerical minimization over the two independent real parameters θ and ϕ.

3. The system in noisy environment

We consider a composite system of a qubit A and a qutrit B such that the two marginal systems are locally or multilocally coupled to their environments. The local and multilocal couplings describe the situations when either the qubit or the qutrit or both the qubit and the qutrit are independently influenced by their own environments.

The interaction of a system with an environment is studied in terms of various quantum channels (noise) such as phase damping noise and amplitude damping noise. When the density matrix of a system is influenced by a dephasing noise, the diagonal elements of the density matrix remain unaffected while the off-diagonal elements decay. The dynamics of quantum correlations of the system we have chosen for the present studies have been investigated in Ref. [36] under the condition that only the qutrit is coupled locally to dephasing environment. In the first part of our study, we reconsider the effect of dephasing noise on the same system, however, in multilocal coupling and show that such coupling of the system with dephasing noise can result in generation of quantum discord and also can leave it noninvariant. The second part of our study explores the influence of amplitude damping noise on the dynamics of quantum correlations. It is shown that for a finite time both local and multilocal coupling with amplitude damping noise generate quantum discord, however, completely destroys it in the asymptotic limit. The easy way of studying the dynamics of an open quantum system is to use Kraus operator formalism. The Kraus operators for a single qubit and single qutrit dephasing noise are, respectively, given as

$$E^D_{A0} = \mathrm{diag}(1, \sqrt{1-\gamma_A(t)}), \quad E^D_{A1} = \mathrm{diag}(0, \sqrt{\gamma_A(t)}), \tag{6}$$