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Impulsive synchronization of Lü chaotic systems via the hybrid controller

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ABSTRACT

In this paper, the impulsive synchronization problem of Lü chaotic systems is investigated by using a novel hybrid controller. The proposed hybrid controller is composed of a single controller and impulsive controller. Using the impulsive theory and the novel hybrid controller, some sufficient conditions are derived for the synchronization of Lü chaotic systems. Numerical simulation example is provided to verify the effectiveness of the proposed approach. The simulation results show that the proposed control scheme has a fast convergence rate compared with the conventional single controller method and impulsive controller method.

1. Introduction

In the past years, different synchronization schemes have been proposed for achieving the synchronization of chaotic system, such as linear and nonlinear feedback synchronization [1,2], adaptive synchronization [3–6], observer based control method [7,8], and so on.

Recently, similar to pinning control of complex dynamical networks, different control schemes of the chaotic system have been proposed by controlling one component of the chaotic system, that is, the proposed controller is a single controller [9–16]. Among the existing publications, some papers focused on 'pinning control' on chaotic systems by the idea of linear feedback control or adaptive control. On the other hand, impulsive control strategies have been widely used to synchronize coupled chaotic dynamical systems due to their potential advantages over general continuous control schemes [17–24]. What is more, the impulsive controller usually has a relatively simple structure. Its necessity and importance lie in that, in some cases, the system cannot be controlled by continuous control. Additionally, impulsive control may give a more efficient method to deal with systems that cannot endure continuous disturbance. Furthermore, impulsive method can also greatly reduce the control cost.

It is worthwhile pointing out that in most recent results appearing in the literature dealing with the stabilization or synchronization of chaotic systems by impulsive control or linear and nonlinear feedback control. In this paper, the impulsive synchronization problem of Lü chaotic systems is investigated by using a novel hybrid controller. The proposed hybrid controller is composed of a single controller and impulsive controller, the proposed hybrid control scheme has faster convergence rate.

This work is organized as follows: Section 2 gives the theoretical analyses. In Section 3, numerical example is used to show the implementation of the proposed scheme. Section 4 gives the conclusion of the paper.

2. Theoretical analyses

Suppose the chaotic system in general form as follows

$$\dot{x} = f(t, x), \tag{1}$$

where $x = (x_1, x_2, \ldots, x_n)^T \in R^n$, $f(x) = (f_1(x), \ldots f_n(x))^T : R_+ \times R^n \to R^n$ is continuous. Suppose that a discrete instant set $\{t_k\}$ satisfies

$$t_0 < t_1 < \cdots t_k < \cdots, \lim_{k\to\infty} t_k = \infty.$$

Let

$$\Delta x|_{t=t_k} x(t_k^+) - x(t_k) = I_k(x),$$

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where $x(t_k^+) = \lim_{t \to t_k^+} x(t)$, $x(t_k) = \lim_{t \to t_k^-} x(t) = x(t_k^-)$, then an impulsive system is given by

$$\begin{cases} \dot{x} = f(t, x), & t \neq t_k, \\ \Delta x = B_k x, & t = t_k, \\ x(t_0^+) = x_0, & k = 1, 2, \ldots, \end{cases} \tag{2}$$

Now, we consider bidirectionally coupled impulsive control of Lü chaotic systems [25] in the form as follows:

$$\begin{cases} \dot{x}_1 = a(x_2 - x_1), & t \neq t_k, \\ \dot{x}_2 = -x_1 x_3 + c x_2, & t \neq t_k, \\ \dot{x}_3 = x_1 x_2 - b x_3, & t \neq t_k, \\ \Delta x = B_k(x - y) = -B_k e, & t = t_k, \end{cases} \tag{3}$$

$$\begin{cases} \dot{y}_1 = a(y_2 - y_1), & t \neq t_k, \\ \dot{y}_2 = -y_1 y_3 + c y_2 + u, & t \neq t_k, \\ \dot{y}_3 = y_1 y_2 - b y_3, & t \neq t_k, \\ \Delta y = B_k(y - x) = B_k e, & t = t_k, \end{cases} \tag{4}$$

where $(a, b, c) = (36, 20, 3)$, the single controller $u = -k_1 e_1 - k_2 e_2 - k_3 e_3$, $k_i \in R$, $i = 1, 2, 3$, then the error impulsive systems as follows.

$$\begin{cases} \dot{e}_1 = a(e_2 - e_1), & t \neq t_k, \\ \dot{e}_2 = -y_1 e_3 - x_3 e_1 + c e_2 - k_1 e_1 - k_2 e_2 - k_3 e_3, & t \neq t_k, \\ \dot{e}_3 = y_1 e_2 + x_2 e_1 - b e_3, & t \neq t_k, \\ \Delta e = 2 B_k e, & t = t_k, \end{cases} \tag{5}$$

where $e = (e_1, e_2, e_3)^T = (y_1 - x_1, y_2 - x_2, y_3 - x_3)^T$.

The objective is to find some conditions on the control gains, B_k and the impulsive distances $t_k - t_{k-1}$, $k = 1, 2, \ldots$, such that the impulsive system (5) is asymptotical stable at origin.

Theorem 1. If there exists two constants $\theta \geq 1$, $\mu > 0$ and $\eta_k = \lambda_{\max}[(I + 2B_k)^T Q (I + 2B_k) Q^{-1}]$, then systems (3) and (4) can realize impulsive synchronization using the following form (1)–(3):

(1) $\begin{pmatrix} -\alpha & 0 & 0 \\ 0 & -\beta & 0 \\ 0 & 0 & -\gamma \end{pmatrix} < 0$

(2) $\begin{pmatrix} -2a\alpha - \mu\alpha & a\alpha + \beta\left|X_3\right| - \beta k_1 & \gamma\left|X_2\right| \\ \Delta & 2c\beta - 2\beta k_2 - \mu\beta & (\beta + \gamma)\left|Y_1\right| - \beta k_3 \\ \Delta & \Delta & -2b\gamma - \mu\gamma \end{pmatrix} < 0$

(3) $\ln \theta \eta_k + \mu(t_k - t_{k-1}) \leq 0$

where $Q = diag(\alpha, \beta, \gamma)$, $\alpha, \beta, \gamma \in R^+$, $\left|X_2\right|$, $\left|X_3\right|$ and $\left|Y_1\right|$ are the upper bounds of the absolute values of the states x_2, x_3 and y_1, Δ denotes the symmetric terms.

Proof. Let the candidate Lyapunov function be in the form of

$V(e) = e^T Q e = \alpha e_1^2 + \beta e_2^2 + \gamma e_3^2.$

The time derivative along the trajectory (5) is:

$$\begin{aligned} \dot{V}(e) &= 2\alpha \dot{e}_1 e_1 + 2\beta \dot{e}_2 e_2 + 2\gamma \dot{e}_3 e_3 = 2\alpha e_1 [a(e_2 - e_1)] \\ &+ 2\beta e_2 [c e_2 - x_3 e_1 - y_1 e_3 - k_1 e_1 - k_2 e_2 - k_3 e_3] \\ &+ 2\gamma e_3 (y_1 e_2 + x_2 e_1 - b e_3) = 2a\alpha e_1 e_2 - 2a\alpha e_1^2 + 2c\beta e_2^2 \\ &- 2\beta y_1 e_2 e_3 - 2\beta x_3 e_1 e_2 - 2\beta k_1 e_1 e_2 - 2\beta k_2 e_2^2 - 2\beta k_3 e_2 e_3 \\ &+ 2\gamma y_1 e_2 e_3 + 2\gamma x_2 e_1 e_3 - 2\gamma b e_3^2 \leq 2a\alpha e_1 e_2 - 2a\alpha e_1^2 \\ &+ 2c\beta e_2^2 + 2\beta\, |y_1|\, e_2 e_3 + 2\beta\, |x_3|\, e_1 e_2 - 2\beta k_1 e_1 e_2 - 2\beta k_2 e_2^2 \\ &- 2\beta k_3 e_2 e_3 + 2\gamma\, |y_1|\, e_2 e_3 + 2\gamma\, |x_2|\, e_1 e_3 - 2\gamma b e_3^2 \leq 2a\alpha e_1 e_2 \\ &- 2a\alpha e_1^2 + 2c\beta e_2^2 + 2\beta\, |y_1|\, e_2 e_3 + 2\beta\, |x_3|\, e_1 e_2 - 2\beta k_1 e_1 e_2 \\ &- 2\beta k_2 e_2^2 - 2\beta k_3 e_2 e_3 + 2\gamma\, |y_1|\, e_2 e_3 + 2\gamma\, |x_2|\, e_1 e_3 \\ &- 2\gamma b e_3^2 \leq 2a\alpha e_1 e_2 - 2a\alpha e_1^2 + 2c\beta e_2^2 + (2\beta + 2\gamma)\left|Y_1\right| e_2 e_3 \\ &+ 2\beta\left|X_3\right| e_1 e_2 - 2\beta k_1 e_1 e_2 - 2\beta k_2 e_2^2 - 2\beta k_3 e_2 e_3 \\ &+ 2\gamma\left|X_2\right| e_1 e_3 - 2\gamma b e_3^2 = \mu e^T \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} e \\ &+ e^T \begin{pmatrix} -2a\alpha - \mu\alpha & a\alpha + \beta\left|X_3\right| - \beta k_1 & \gamma\left|X_2\right| \\ \Delta & 2c\beta - 2\beta k_2 - \mu\beta & (\beta + \gamma)\left|Y_1\right| - \beta k_3 \\ \Delta & \Delta & -2b\gamma - \mu\gamma \end{pmatrix} \\ &\times e \leq \mu e^T \begin{pmatrix} \alpha & 0 & 0 \\ 0 & \beta & 0 \\ 0 & 0 & \gamma \end{pmatrix} e = \mu e^T Q e = \mu V(e(t)). \end{aligned}$$

This implies that

$V(e(t)) \leq V(e(t_{k-1}^+)) \exp(\mu(t - t_{k-1}))$, $t \in (t_{k-1}, t_k]$, $k = 1, 2, \ldots$

Now from (5), we have

$$\begin{aligned} V(e(t_k^+)) &= [(I + 2B_k)e]^T Q (I + 2B_k) e \\ &= e^T [(I + 2B_k)^T Q (I + 2B_k) Q^{-1}] Q e \leq \eta_k e^T Q e = \eta_k V(e(t_k)). \end{aligned}$$

When $t \in (t_0, t_1]$, $V(e(t)) \leq V(e(t_0^+)) \exp(\mu(t - t_0))$, then

$V(e(t_1)) \leq V(e(t_0^+)) \exp(\mu(t_1 - t_0))$.

So,

$V(e(t_1^+)) \leq \eta_1 V(e(t_1)) \leq \eta_1 V(e(t_0^+)) \exp(\mu(t_1 - t_0))$.

In the same way for $t \in (t_1, t_2]$, we have

$V(e(t)) \leq V(e(t_1^+)) \exp(\mu(t - t_1)) \leq \eta_1 V(e(t_0^+)) \exp(\mu(t - t_0))$

In general for any $t \in (t_k, t_{k+1}]$, one finds that

$V(e(t)) \leq V(e(t_0^+)) \eta_1 \eta_2 \cdots \eta_k \exp(\mu(t - t_0))$.

Thus for $\forall t \in (t_k, t_{k+1}]$, $k = 1, 2, \ldots$, we have

$$\begin{aligned} V(e(t)) &\leq V(e(t_0^+)) \eta_1 \eta_2 \cdots \eta_k \exp(\mu(t - t_0)) \leq V(e(t_0^+)) \eta_1 \eta_2 \cdots \eta_k \\ &\exp(\mu(t_{k+1} - t_0)) = V(e(t_0^+)) \eta_1 \exp(\mu(t_2 - t_1)) \eta_2 \\ &\exp(\mu(t_3 - t_2)) \cdots \eta_k \exp(\mu(t_{k+1} - t_k)) \exp(\mu(t - t_0)). \end{aligned}$$

From the assumptions given in the theorem $\eta_k \exp(\mu(t_{k+1} - t_k)) \leq \frac{1}{\theta}$, $k = 1, 2, \ldots$, we have

$$V(e(t)) \leq V(e(t_0^+)) \frac{1}{\theta^k} \exp(\mu(t - t_0)).$$

That is $V(e(t)) \leq V(e(t_0^+))(1/\theta^k) \exp(\mu(t - t_0))$, $t \geq t_0$.

When $\theta \geq 1$, from Ref. [26], this implies that the origin in system (5) is globally asymptotically stable or the slave system is synchronized with the master system asymptotically for any initial conditions. By this we conclude proof of the theorem.