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# Dark soliton and periodic wave solutions of the Biswas–Milovic equation

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ABSTRACT

By means of the idea of sine–cosine method and He's semi-inverse method, some analytic solutions for the Biswas–Milovic equation are presented. The sine–cosine method and He's semi-inverse method are used to construct exact solitary solutions of this equation. Biswas–Milovic equation is a generalized version of the familiar nonlinear Schrodinger's equation describing the propagation of solitons through optical fibers for trans-continental and transoceanic distances. New families of exact travelling wave solutions of the Biswas–Milovic equation are successfully obtained.

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## 1. Introduction

The theory of optical solitons has been one of the widely popular topics of research for the last few decades [1–6]. In order to model the dynamics of optical soliton propagation for trans-oceanic and trans-continental distances, an improved model is the Biswas–Milovic equation (BME) that was first introduced in 2010 [1]. This model accounts for the departure from perfection for the dynamics of soliton propagation through optical fibers. There are several detrimental effects that are inevitable. These include the errors due to the imperfections of the cylindrical geometry of the fibers, randomness of the injection of the pulses at the initial end of the fiber and others. Therefore the group velocity dispersion, evolution of the pulses will not be quite governed by the NLSE. Instead a generalized version of the NLSE, namely the Biswas–Milovic equation is a model that is closer to reality [2]. The Biswas–Milovic equation that is going to be studied in this paper is given by [3]

$$i(q^m)_t + a(q^m)_{xx} + bF(|q|^2)q^m = 0. \tag{1}$$

where $i=\sqrt{-1}$ and the dependent variable $q$ is a complex valued function, while $x$ and $t$ are the two independent variables. The coefficients $a$ and $b$ are constants where $ab>0$. The parameter $m1$.

Eq. (1) is a nonlinear partial differential equation that is not integrable, in general. The non-integrability is not necessarily related to the nonlinear term in it. Also, in (1), $F$ is a real-valued algebraic function and it is necessary to have smoothness of the complex function $F(|q|^2): C \mapsto C$. Considering the complex plane $C$ as a two-dimensional linear space $R^2$, the function $F(|q|^2)$ is $k$ times continuously differentiable, so [4]

$$F(|q|^2) \in \bigcup_{m,n=1}^{\infty} C^k((-n,n)\times(-m,m);R^2). \tag{2}$$

In this paper we will study the case $m=1$ of Biswas–Milovic equation

$$iq_t + aq_{xx} + bF(|q|^2)q = 0. \tag{3}$$

In order to seek exact solutions of Eq. (3), we assume that $q(x, 0)=e^{ix}$ and in this case the Kerr law of nonlinearity appears in nonlinear optics [5] is

$$F(s) = s. \tag{4}$$

so Eq. (3) becomes

$$i\,q_t + a\,q_{xx} + b\,|q|^2\,q = 0. \tag{5}$$

Now, in order to seek exact solutions of Eq. (5), we assume

$$q(x,t) = u(\xi) \quad e^{i\theta}, \quad \theta = \alpha x + \beta t, \quad \xi = x - 2a\alpha t, \tag{6}$$

where $\alpha$, $\beta$ are constants to be determined later. Substituting Eq. (6) into Eq. (3), we have

$$-(\beta + a\alpha^2)u(\xi) + au''(\xi) + bu^3(\xi) = 0, \tag{7}$$

The paper is prepared as follows: In Sections 2 and 3, the He's semi-inverse method and sine–cosine method are discussed; In Section 4 and 5, we exert these methods to the Biswas–Milovic equation. Finally, the paper is concluded in Section 6.

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## 2. Description of He's semi-inverse method

We suppose that the given nonlinear partial differential equation for to be in the form

$$P(u, u_x, u_t, u_{xx}, u_{tt}, u_{xt}, \ldots) = 0, \tag{8}$$

where $P$ is a polynomial in its arguments. The essence of He's semi-inverse method can be presented in the following steps:

*Step 1.* Seek solitary wave solutions of (8) by taking $u(x, t) = U(\xi)$, $\xi = x - ct$ and transform (8) to the ordinary differential equation

$$U(u, u', u'', \ldots) = 0, \tag{9}$$

where prime denotes the derivative with respect to $\xi$.

*Step 2.* If possible, integrate (9) term by term one or more times. This yields constant(s) of integration. For simplicity, the integration constant(s) can be set to zero.

*Step 3.* According to He's semi-inverse method, we construct the following trial-functional

$$J(U) = \int L\, d\xi, \tag{10}$$

where $L$ is an unknown function of $U$ and its derivatives.

There exist alternative approaches to the construction of the trial-functionals, see Refs. [6,7].

*Step 4.* By the Ritz method, we can obtain different forms of solitary wave solutions, in the form

$$U(\xi) = p\,\mathrm{sech}^n(q\xi), \tag{11}$$

where $P$ and $q$ are constants to be further determined.

Substituting (11) into (10) and making $J$ stationary with respect to $P$ and $q$ results in

$$\frac{\partial J}{\partial p} = 0, \tag{12}$$

$$\frac{\partial J}{\partial q} = 0. \tag{13}$$

Solving simultaneously (12) and (13) we obtain and. Hence, the solitary wave solution (11) is well determined.

## 3. Description of sine–cosine method

1. We introduce the wave variable $\xi = x - ct$ into the PDE

$$P(u, u_t, u_x, u_{tt}, u_{xx}, u_{tx}, \ldots) = 0, \tag{14}$$

where $u(x, t)$ is traveling wave solution. This enables us to use the following changes:

$$\frac{\partial}{\partial t} = -c\,\frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial t^2} = c^2\,\frac{\partial^2}{\partial \xi^2}, \quad \frac{\partial}{\partial x} = \frac{\partial}{\partial \xi}, \quad \frac{\partial^2}{\partial x^2} = \frac{\partial^2}{\partial \xi^2}, \cdots. \tag{15}$$

One can immediately reduce the nonlinear PDE (14) into a nonlinear ODE

$$Q(u, u_\xi, u_{\xi\xi}, u_{\xi\xi\xi}, \ldots) = 0. \tag{16}$$

The ordinary differential equation (16) is then integrated as long as all terms contain derivatives, where we neglect integration constants.

2. The solutions of many nonlinear equations can be expressed in the form [8]

$$u(x, t) = \begin{cases} \lambda \sin^m(\mu\,\xi), & |\xi| \le \dfrac{\pi}{\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{17}$$

or in the form

$$u(x, t) = \begin{cases} \lambda \cos^m(\mu\,\xi), & |\xi| \le \dfrac{\pi}{2\mu}, \\ 0 & \text{otherwise}, \end{cases} \tag{18}$$

where $\lambda$, $\mu$ and $m \neq 0$ are parameters that will be determined, $\mu$ and $c$ are the wave number and the wave speed respectively. We use

$$\begin{aligned} &u(\xi) = \lambda \sin^m(\mu\xi), \\ &u^n(\xi) = \lambda^n \sin^{nm}(\mu\xi), \\ &(u^n)_\xi = n\mu\, m\,\lambda^n \cos(\mu\xi)\sin^{nm-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\, m^2\lambda^n \sin^{nm}(\mu\xi) + n\mu^2\lambda^n m(nm-1)\sin^{nm-2}(\mu\xi), \end{aligned} \tag{19}$$

and the derivatives of (18) becomes

$$\begin{aligned} &u(\xi) = \lambda \cos^m(\mu\,\xi), \\ &u^n(\xi) = \lambda^n \cos^{n\,m}(\mu\,\xi), \\ &(u^n)_\xi = -n\mu\, m\,\lambda^n \sin(\mu\xi)\cos^{n\,m-1}(\mu\xi), \\ &(u^n)_{\xi\xi} = -n^2\mu^2\, m^2\lambda^n \cos^{n\,m}(\mu\xi) + n\mu^2\lambda^n\, m(n\,m-1)\cos^{n\,m-2}(\mu\xi), \end{aligned} \tag{20}$$

and so on for other derivatives.

3. We substitute (19) or (20) into the reduced equation obtained above in (16), balance the terms of the cosine functions when (20) is used, or balance the terms of the sine functions when (19) is used, and solving the resulting system of algebraic equations by using the computerized symbolic calculations. We next collect all terms whit same power in $\cos^k(\mu\xi)$ or $\sin^k(\mu\xi)$ and set to zero their coefficients to get a system of algebraic equations among the unknowns $\mu$, $m$ and $\lambda$. We obtained all possible value of the parameters $\mu$, $m$ and $\lambda$ [9].

## 4. Application of He's semi-inverse method

According to Ref. [6], by He's semi-inverse method [7], we can arrive at the following variational formulation:

$$J(v) = \int_0^\infty \left[ -\frac{a}{2}(u\prime)^2 - \frac{(\beta + a\alpha^2)}{2}u^2 + \frac{b}{4}u^4 \right] d\xi. \tag{21}$$

We assume the solitary solution in the following form:

$$u(\xi) = p\,\mathrm{sech}(q\xi) \tag{22}$$

where $p$, $q$ are unknown constants to be further determined.

By substituting (22) into (21) we obtain

$$\begin{aligned} J &= \int_0^\infty \left[ \left( \frac{1}{2}p^2aq^2 + \frac{1}{4}bp^4 \right) \mathrm{sech}^4(q\xi) \right] d\xi \\ &\quad + \int_0^\infty \left[ \left( -\frac{1}{2}p^2aq^2 - \frac{1}{2}p^2\beta - \frac{1}{2}p^2\alpha^2 a \right) \mathrm{sech}^2(q\xi) \right] d\xi \\ &= \frac{1}{q}\left( \frac{1}{2}p^2aq^2 + \frac{1}{4}bp^4 \right) \int_0^\infty \mathrm{sech}^4(\theta)\, d\theta \\ &\quad + \frac{1}{q}\left( -\frac{1}{2}p^2aq^2 - \frac{1}{2}p^2\beta - \frac{1}{2}p^2\alpha^2 a \right) \int_0^\infty \mathrm{sech}^2(\theta)\, d\theta \\ &= \frac{2}{3q}\left( \frac{1}{2}p^2aq^2 + \frac{1}{4}bp^4 \right) + \frac{1}{q}\left( -\frac{1}{2}p^2aq^2 - \frac{1}{2}p^2\beta - \frac{1}{2}p^2\alpha^2 a \right). \end{aligned}$$