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Comparative studies of properties of standard and elegant Laguerre–Gaussian beams

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ABSTRACT

The properties of standard Laguerre–Gaussian beam (sLGB) and elegant Laguerre–Gaussian beam (eLGB) through the misaligned complex hard edge apertures (MCHEAs) are investigated. Analytical formulae for the far field distributions of sLGBs and eLGBs through the different hard edge apertures are derived based on the generalized Huygens–Fresnel diffraction integral equation. The propagation properties and beam qualities of sLGBs and eLGBs are studied comparatively. It is shown that the divergence angle of sLGBs with the low order modes is less than that of eLGBs. However, the energy distributions of eLGBs with the high order modes after through the multiple hard edged apertures are more convergent in comparison with sLGBs, and eLGBs have the better beam quality than sLGBs. The study results are useful for the propagations and applications of sLGBs and eLGBs through the complex optical systems.

1. Introduction

It is well known that the standard Laguerre–Gaussian beam (sLGB) is the eigenmode of the paraxial wave equation [1–3]. In the past decades, sLGBs have been paid enormous attention due to the extensive applications in free-space communications, electron acceleration, nanoparticles trapping, atom interferometers, atom guiding and atom trapping [4,5]. There are many methods for obtaining sLGB, such as, spatial light modulators [6], mode selection [7], through the conversion of Hermite–Gaussian beams by an astigmatic mode converter [8], and so on. The propagation properties of sLGB through the paraxial ABCD optical systems have been reported [9–11]. An elegant Laguerre–Gaussian beam (eLGB) was proposed as an extension of the sLGB [2]. Both eLGB and sLGB satisfy the paraxial wave equation, while the eLGB has the more symmetrical form in comparison with the sLGB [1–3]. The former contains the polynomials with a complex argument, but in the latter the argument is real. Some authors have tried to study the properties of eLGB through arbitrary optical systems [12–20], such as, the evolution of eLGB [12], Kurtosis parameter [13], vortex structure of eLGB [14], paraxial ABCD optical system [15], turbulent atmosphere [16], non-Kolmogorov turbulence [17], misaligned optical system [20], annular aperture [21,22], a sequence of apertures [23], misaligned paraxial optical systems [24–29] and so on. However, propagations of sLGBs and eLGBs could encounter the complex misaligned optical systems, for example, the misaligned multiple hard edge apertures. Up to now, to our knowledge, the propagation properties of sLGBs and eLGBs through the misaligned multiple hard edge apertures haven't been reported.

In this paper, our aim is to study the propagations of sLGBs and eLGBs through the misaligned multiple hard edge apertures. Then the properties and beam qualities of sLGBs and eLGBs are studied numerically and comparatively. The analytical formulae for the far field distributions of sLGBs and eLGBs through the misaligned multiple hard edge apertures are derived. The beam quality factors [30–33] of sLGBs and eLGBs are studied comparatively. The results can afford references for studying deeply Laguerre–Gaussian beams (LGBs).

2. Propagation formulae of sLGBs and eLGBs through MCHEAs

The electric field of LGB (sLGB or eLGB) in the plane of the source $z=0$, is expressed in cylindrical coordinate system as follows [13–16]

$$\varepsilon_0(r_0,\theta_0,0)=\left(\frac{r_0}{a_0}\right)^m L_n^m\left(\frac{r_0^2}{a_0^2}\right)\exp\left(-\frac{r_0^2}{w_0^2}\right)\exp(-im\theta_0). \quad (1)$$

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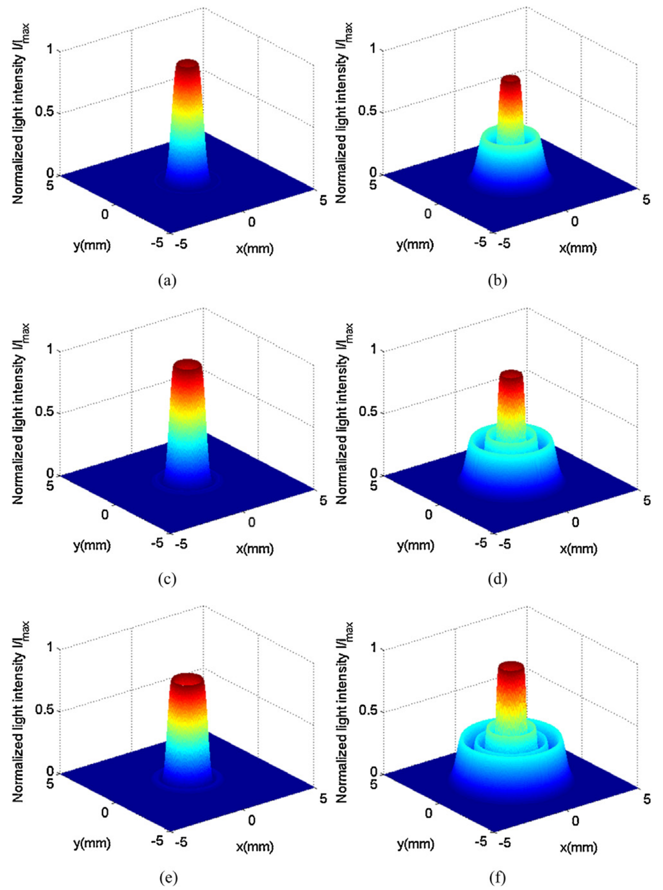

Fig. 1. Normalized intensity distribution of eLGB and sLGB in the plane of source $z=0$ for the different values of mode orders m and n with $w_0 = 1$ mm and $\lambda = 10.6$ μm, (a) $m=n=1$, for eLGB, (b) $m=n=1$, for sLGB, (c) $m=n=2$, for eLGB, (d) $m=n=2$, for sLGB, (e) $m=n=3$, for eLGB, (f) $m=n=3$, for sLGB.

where r_0 and θ_0 are the radial coordinate and azimuthal coordinate, respectively, $L_n^m(\cdot)$ is the Laguerre polynomials with mode orders m and n, w_0 is the radius of beam waist, sLGB and eLGB are described by Eq. (1) for $a_0 = w_0/\sqrt{2}$ and $a_0 = w_0$, respectively. The near field distributions for sLGB and eLGB are shown in Fig. 1. Where Fig. 1(a, c, e) and Fig. 1(b, d, f) denote the light intensities of eLGB and sLGB, respectively. It is found that the central dark areas for sLGB and eLGB will become large with the increase of order m. However, the energy distributions of eLGB are centralized in comparison with sLGB. The diagram of the misaligned optical system [24–29] with multiple hard edge apertures is shown in Fig. 2, where RP_i and $RP_{im}(i=1, 2)$ are aligned and misaligned reference planes, respectively. The distance between RP_1 and RP_2 is $(p-1)l$, the line deviation and angle deviation at the different apertures are τ_p and τ_p', respectively.

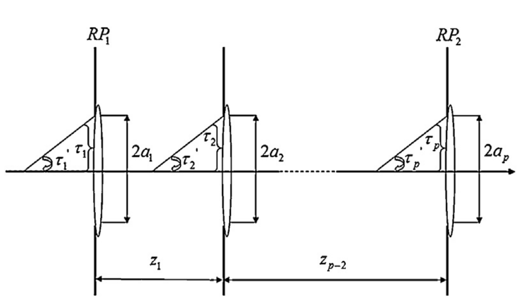

Fig. 2. Geometrical optical pathway diagram of sLGB and eLGB through the misaligned optical system with multiple hard edge apertures.

The propagations of sLGB and eLGB through MCHEAs are characterized by generalized Huygens-Fresnel diffraction integral equation as follow [13–15]

$$\varepsilon_p(r_p,\theta_p,z_p) = \int_0^{2\pi}\int_0^{a_p} \varepsilon_{p-1}(r_{p-1},\theta_{p-1},0)\exp\left\{-\frac{ik}{2B_p}[A_p r_{p-1}^2 - 2r_p r_{p-1}\cos(\theta_p-\theta_{p-1}) + D_p r_p^2 + e_p r_{p-1}\cos\theta_{p-1} + f_p r_{p-1}\sin\theta_{p-1} + g_p r_p\cos\theta_p + h_p r_p \sin\theta_p]\right\} \times r_{p-1}dr_{p-1}d\theta_{p-1}. \quad (2)$$

where $p=1, 2, 3\ldots$, $k=2\pi/\lambda$ is the wave number, a_p is the radius of the pth hard edge aperture, A_p, B_p, C_p and D_p are the propagation matrix elements, and parameters e_p, f_p, g_p and h_p are given by

$$e_p = 2[\alpha_{Tp}\tau_p\cos\varphi_p + \beta_{Tp}\arctan(\cos\varphi_p\tan\tau_p')]. \quad (3)$$

$$f_p = 2[\alpha_{Tp}\tau_p\sin\varphi_p + \beta_{Tp}\arctan(\sin\varphi_p\tan\tau_p')]. \quad (4)$$

$$g_p = 2(B_p\gamma_{Tp} - D_p\alpha_{Tp})\tau_p\cos\varphi_p + 2(B_p\delta_{Tp} - D_p\beta_{Tp})\arctan(\cos\varphi_p\tan\tau_p'). \quad (5)$$

$$h_p = 2(B_p\gamma_{Tp} - D_p\alpha_{Tp})\tau_p\sin\varphi_p + 2(B_p\delta_{Tp} - D_p\beta_{Tp})\arctan(\sin\varphi_p\tan\tau_p'). \quad (6)$$

where φ_p is the misaligned azimuth angle, τ_p and τ_p' are the misaligned line deviation and angle deviation, respectively. The misaligned matrix elements satisfy the following expressions:

$$\alpha_{Tp} = 1 - A_p, \quad \beta_{Tp} = l - B_p, \quad \gamma_{Tp} = -C_p, \quad \delta_{Tp} = 1 - D_p. \quad (7)$$

Introducing the hard aperture function [21–23]

$$T_p(r_{p-1}) = \begin{cases} 1 & (|r_{p-1}| \le a_p) \\ 0 & (|r_{p-1}| > a_p) \end{cases}. \quad (8)$$

Generally, the hard aperture function should be expanded into a finite sum of complex Gaussian functions

$$T_p(r_{p-1}) = \sum_{h_p=1}^{N} A_{h_p}\exp\left(-\frac{B_{h_p}}{{a_p}^2}r_{p-1}^2\right). \quad (9)$$

where A_{hp} and B_{hp} are the expansion and Gaussian coefficients, respectively.

Substituting Eqs. (1) and (9) into Eq. (2), and then let $p=1$, the propagation formula of sLGB and eLGB through the first hard-edge aperture can be expressed as follow

$$\varepsilon_1(r_1,\theta_1,z_1) = \frac{ik\exp(-ikz_1)}{2\pi B_1}\exp\left[-\frac{ik}{2B_1}(D_1r_1^2 + g_1r_1\cos\theta_1 + h_1r_1\sin\theta_1)\right] \times \sum_{h_1=1}^{N}A_{h1}\int_0^{2\pi}\int_0^{\infty}\exp\left(-\frac{B_{h1}}{{a_1}^2}r_0^2\right)\left(\frac{r_0}{a_0}\right)^m L_n^m\left(\frac{r_0^2}{a_0^2}\right)\exp\left(-\frac{r_0^2}{w_0^2}\right)\exp(-im\theta_0)\times exp\left\{-\frac{ik}{2B_1}[A_1r_0^2 - 2r_1r_0\cos(\theta_1-\theta_0) + e_1r_0\cos\theta_0 + f_1r_0\sin\theta_0]\right\} \times r_0dr_0d\theta_0. \quad (10)$$