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Projective synchronization for a fractional-order chaotic system via single sinusoidal coupling

Li Chun-Lai*, Zhang Mei*, Zhou Feng, Yang Xuan-Bing

College of Physics and Electronics, Hunan Institute of Science and Technology, Yueyang 414006, China

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ABSTRACT

This paper investigates the sufficient criterion for robust projective synchronization of a fractional-order chaotic system. On the basis of the input-to-state stable (ISS) theory, a single sinusoidal state coupling controller is derived to achieve projective synchronization of this fractional dynamical system by considering bounded interference. The control parameters are obtained by resolving the matrix inequality into some algebraic inequalities. Numerical simulations are presented to verify the effectiveness of the introduced synchronization scheme.

1. Introduction

Fractional calculus is a classical notion in the field of applied mathematics, which can be regarded as a generalization dealing with the differentiation and integration of arbitrary non-integer order [1,2]. In comparison with classical integer-order model, the distinguished feature of fractional-order system is that it has infinite memory, which can be utilized to characterize different nonlinear phenomena more accurately [3,4]. In addition, due to the fact that fractional system holds more adjustable variables, it can enlarge the key space and thus be applied in encryption more safely [5–7]. Therefore, the dynamical analysis and synchronization of fractional order chaotic system is an important topic of study in both research and application.

The dynamical feature of projective synchronization is the state trajectories of the master system verge on those of slave system up to a constant proportional factor. This property can be used to extend binary digital communication to N-nary one with shorter response time. Projective synchronization of integer-order chaotic system has been investigated thoroughly [8–12], including chaotic communication [8] and designing of Hopf limit circle [12]. However, the study on projective synchronization of fractional-order dynamical system is less in number. Its unpopularity may be due to most of the stability theories and methods on synchronization for the integer-order chaotic system cannot be extended to the fractional-order simply [5,13]. Although there are some works on projective synchronization of fractional system [14–17], it should be observed that these control schemes are complicated and of expensive control cost, and that they have not considered the practical circumstances with external noise disturbances. Therefore, we not only require master-slave systems to be synchronized, but also to have the property of input-to-state stability.

Synchronization schemes of integer-order chaotic system via sinusoidal state feedback have been discussed numerically and analytically [18,19]. The main advantage of a sinusoidal control scheme is that the control input is always smooth and bounded, and that the energy consumption is low. Superiority though the sinusoidal synchronization scheme holds, it has not been investigated in fractional-order dynamical system as far as we know.

In this work, we concentrate on the fractional version of a three-dimensional autonomous chaotic system with amplitude modulation and constant Lyapunov spectrum [20]. By considering the fact that the amplitude parameters can provide the scale factors, a projective

* Corresponding authors. Tel.: +86 7308640052.
E-mail addresses: ChaoEncryption@126.com (L. Chun-Lai), MeiZhang163163@163.com (Z. Mei).

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synchronization scheme of the fractional system with external stochastic noise is studied. First, based on the ISS theory of fractional system [13], the projective synchronization criterion of two identical fractional chaotic systems coupled by single sinusoidal state error is derived in the form of a matrix inequality, in which the control parameters can be evaluated by resolving the matrix inequality into some algebraic inequalities. Then, to obtain the control parameters more expediently, the state error variable is restricted in a sub-region. Numerical simulations are shown to further verify the feasibility of the presented synchronization scheme.

2. Model of fractional-order system

2.1. Preliminaries of fractional calculus

Some preliminaries such as definition, basic property and stability theory of fractional calculus are recalled in this section, which will be used in the sequel.

There exist multiple definitions of fractional derivative, in our paper only the Caputo fractional-order derivative is adopted for its explicit physical interpretation, as follows

Definition 1 *([21])*. The Caputo fractional-order derivative of function $f(t)$ with respect to t is defined by

$$D^q f(t) = \frac{1}{\Gamma(n-q)} \int_{t_0}^{t} (t-\tau)^{-q+n-1} \left(\frac{\mathrm{d}}{\mathrm{d}\tau}\right)^n f(\tau)\mathrm{d}\tau$$

where q is the order of fractional derivative, n is an integer satisfying $n = [q] + 1$, $[q]$ is the integer part of q, $\Gamma(\cdot)$ is the Gamma function.

Property 1. For Caputo fractional derivative operator, we have $D^q(px_1(t) + qx_2(t)) = p_1 D^q x_1(t) + p_1 D^q x_2(t)$, where p_1, p_2 are real constants.

Lemma 1 ([22]). For the commensurate fractional-order system $D^q X = F(X)$ with $X \in R^n$, $F(X) \in R^n$ and $q \in (0, 1]$, the necessary condition of generating chaos is $q\pi/2 - \min\left\{\left|arg\left(\lambda_i\right)\right|\right\} \geq 0$, where λ_i is the characteristic root of the Jacobian matrix $J = \partial F(X)/\partial X$, $i = 1, 2, \ldots, n$.

Remark 1. In comparison with the numerical scheme of integer order differential equation, the calculation process of fractional differential equation is more complex. In this paper, we employ the predictor-corrector algorithm for fractional differential system. What must be clarified is that the Caputo definition is not an accurate expression of original fractional equation, and the approximation does not take into account all the past of the signals.

2.2. Description of fractional-order chaotic system

In Ref. [20], the authors introduced a chaotic system. Careful analysis shows that, with the increasing of some parameters, the amplitude of the state variables can be adjusted by certain functional relation, yet the Lyapunov exponent spectrums remain invariable. Amplitude controllability of chaotic system is significative in applications. One can obtain the desired signal amplification without any extra circuitry spending and prevent from increasing the probability of failure in circuit operation. Meanwhile, this feature can avoid the influence of the band-limit filter in signal amplification circuit. Therefore, it is deemed to be a promising type system to provide a new security encoding key in chaotic radar and chaotic communication [5,13].

The corresponding fractional version of this system is described as

$$\begin{cases} D^q x_1 = -ax_1 + fx_2 \\ D^q x_2 = bx_2 + gx_1x_3 \\ D^q x_3 = -x_3 - hx_1{}^2 \end{cases} \tag{1}$$

in which $D^q x = \mathrm{d}^q x/\mathrm{d}t^q$ is the fractional derivative in Caputo sense.

When $a = 10$, $b = 6$, $f = 1$, $g = 1$, $h = 1$, fractional system (1) has three equilibrium points and their corresponding characteristic values are

$$P_0(0, 0, 0) : \lambda_1 = -10, \quad \lambda_2 = 6, \quad \lambda_3 = -1$$

$$P_+(7.746, 77.4597, -60) : \lambda_1 = -25.5231, \quad \lambda_2 = 10.2615 + 19.0398\mathrm{i}, \quad \lambda_3 = 10.2615 - 19.0398\mathrm{i}$$

$$P_-(-7.746, -77.4597, -60) : \lambda_1 - 25.5231, \quad \lambda_2 = 10.2615 + 19.0398\mathrm{i}, \quad \lambda_3 = 10.2615 - 19.0398\mathrm{i}$$

It yields $q > \left(2/\pi\right) a\tan\left(\left|Im\left(\lambda*\right)\right| / Re\left(\lambda*\right)\right) = \left(2/\pi\right) a\tan\left(19.0398/10.2615\right) = 0.6853$ from Lemma 1. Therefore, when $q > 0.6853$, the necessary condition for the existence of chaos holds for system (1). The bifurcation diagram with respect to the varying fractional order q is shown in Fig. 1(a), as we know that system (1) behaves chaos when $0.851 \leq q \leq 1$. The corresponding phase portrait and power spectrum are depicted in Fig. 1(b) and (c) with $q = 0.9$.

2.3. Amplitude modulation of fractional-order chaotic system

Theorem 1. For fractional system (1), the dynamic parameters f, g, h can, respectively modulate the amplitude of state variables x_1, x_2, x_3 according to $1/\sqrt{fgh}$, $1/\sqrt{f^3gh}$, $1/(fg)$, and in the meantime the Lyapunov exponent spectrums remain invariable.