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General principles for reflection of laser pulse from a relativistic uniformly-moving mirror

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ABSTRACT

We investigate the reflection of laser pulse from a uniformly moving mirror at relativistic speeds in the general case of inclined mirrors. We use geometrical optics to derive general relations for length, frequency, wavelength and electric field of the pulse after reflection from inclined mirror in the lab frame. In addition, the pulse geometric deformation from reflection is investigated and simulated in two dimensions.

1. Introduction

The discussion of the reflection of light from a uniformly moving mirror is not new [1]. A particular case of the problem was elaborated by Einstein almost a century ago [2]. Einstein considered the oblique incidence of a plane-polarized electromagnetic wave on a perfectly reflecting mirror whose velocity was directed perpendicularly to its surface. Einstein derived the equations for the angle of reflection and the wave characteristics of the reflected light using Lorentz transformations between the reference frame, where the mirror was at rest, and the lab frame. This problem can be solved for the general case in which the mirror is inclined and makes an angle φ with velocity direction. Using Lorentz transformations and a basic principle that the momentum change for reflecting light must be perpendicular to the plane of the mirror [3], or a different approach based on elementary principles of wave optics and the postulates of special relativity [4], the relation between incident and reflected angles can readily be determined for any mirror speed and any particular geometry. Our goal is to derive pulse duration after reflection from an inclined moving mirror and an overall relation. Having such a relation, we can find the maximum compression of the pulse after reflection. Recent advances in ultra-short laser pulses technology have led to an amazing increase in intensity [5]. Experimental and theoretical studies and simulations have shown that under the conditions typical for the interaction of such a high-intensity laser pulses with under-dense plasma or over-dense plasma such as thin layers, the nonlinear characteristics of this interaction results in the formation of electron density modulations in the form of high-density bunches and thin layers moving with a relativistic velocity that will act like relativistic mirrors [6,7]. Achieving the order of femtosecond laser pulses (1 fs = 10^{-15} s) using conventional mode-locking methods in the lasing medium with large frequency bandwidth is known [8], but achieving the order of Attosecond and Zepto-seconds is one of the present century dreams that notable experimental successes herald the arrival of a new branch of science [9]. Applying the relativistic mirror for more compression of short pulses is one of the new methods in reaching to this branch. A great deal of researchers' activities in this branch has been allocated to the methods of producing relativistic mirrors [10]. In this paper we consider the pulse transverse and longitudinal compression in oblique reflection from an inclined relativistic mirror using geometric methods and without the Lorentz transformations.

2. Length and area of the pulse after reflection

In reflection from stationary mirror, according to the ordinary law of reflection of light, the angles of incidence and reflection are equal [11]. When the mirror is moving, but its speed is less than the relativistic limit or by adapting a moving frame in which the mirror is at rest, the ordinary law of reflection is still valid. However, if the mirror speeds up to the relativistic limit, in the stationary lab frame, the ordinary law of reflection is no longer valid. In fact, it is well established that for light reflected from a mirror moving at relativistic speeds, the angle of reflection is not necessarily equal to the angle of incidence [2,12]. Gjurchinovski [4] derived a general relation for the angle of reflection β on the basis of the angle of incidence α and the inclination angle φ of the moving mirror and mirror velocity v in the lab frame (Fig. 3)

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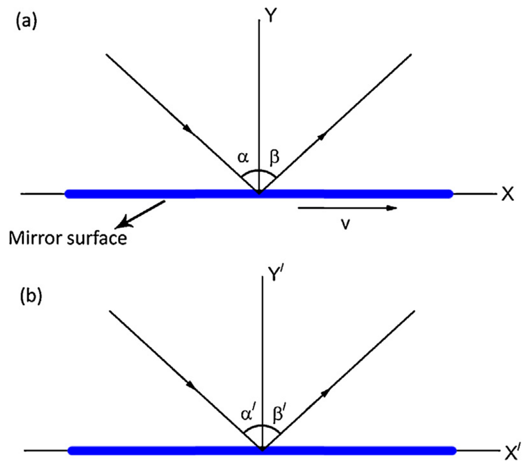

Fig. 1. Reflection of light from the moving mirror in the lab frame (a) and in the moving frame when the mirror is at rest in it (b).

$$\cos\beta = \frac{2\frac{v}{c}\sin\varphi + \left(1+\frac{v^2}{c^2}\sin^2\varphi\right)\cos\alpha}{1+2\frac{v}{c}\sin\varphi\cos\alpha + \frac{v^2}{c^2}\sin^2\varphi}. \tag{1}$$

This relation in the non-relativistic limit $v << c$ (c is the speed of light) will be transformed to the familiar law of reflection,

$$\cos\beta = \cos\alpha. \tag{2}$$

Eq. (1) for $\varphi = 0^\circ$ simplifies to Eq. (2) which means that if the velocity vector is parallel to the mirror surface (or when the mirror is moving in its own direction) the angle of reflection is equal to the angle of incidence and mirror movement has not any effect on the angle of reflection (Fig. 1). This can also be proven by applying Lorentz transformations as follows. For this purpose, we use equations of relativistic aberration of light [13],

$$\tan\theta = \frac{\sin\theta'\sqrt{1-\frac{v^2}{c^2}}}{\cos\theta' + \frac{v}{c}};\ \tan\theta' = \frac{\sin\theta\sqrt{1-\frac{v^2}{c^2}}}{\cos\theta - \frac{v}{c}}, \tag{3}$$

where, θ and θ' determine the orientation of the light ray with respect to the positive X-and X'-axis. These equations can also be expressed as follows:

$$\cos\theta = \frac{\cos\theta' + \frac{v}{c}}{1+\frac{v}{c}\cos\theta'};\ \cos\theta' = \frac{\cos\theta - \frac{v}{c}}{1-\frac{v}{c}\cos\theta} \tag{4}$$

Applying Eq. (4) and Fig. 1, we get

$$\cos\left(\frac{3\pi}{2}+\alpha'\right) = \frac{\cos\left(\frac{3\pi}{2}+\alpha\right) - \frac{v}{c}}{1-\frac{v}{c}\cos\left(\frac{3\pi}{2}+\alpha\right)} \tag{5}$$

which simplifies to,

$$\sin\alpha' = \frac{\sin\alpha - \frac{v}{c}}{1-\frac{v}{c}\sin\alpha}. \tag{6}$$

In the moving frame we have

$$\sin\alpha' = \sin\beta'. \tag{7}$$

Now we can write

$$\cos\left(\frac{\pi}{2}-\beta\right) = \frac{\cos\left(\frac{\pi}{2}-\beta'\right) + \frac{v}{c}}{1+\frac{v}{c}}, \tag{8}$$

which simplifies to,

$$\sin\beta = \frac{\sin\beta' + \frac{v}{c}}{1+\frac{v}{c}\sin\beta'}. \tag{9}$$

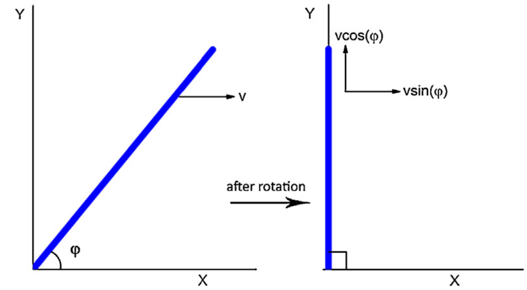

Fig. 2. Frame rotation for changing the inclined mirror to vertical mirror.

Applying Eqs. (6) and (7), we get

$$\sin\beta = \sin\alpha, \tag{10}$$

which shows that, mirror movement has no effect on the angle of reflection in this case.

The authors of reference [3] derived the following equations by using Lorentz transformations,

$$\sin\alpha = \frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\varphi'}} \times \frac{\sin\alpha' + \frac{v}{c}\cos\varphi'}{\sqrt{\gamma^2\left[\sin(\alpha'-\varphi') + \frac{v}{c}\right]^2 + \cos^2(\alpha'-\varphi')}} \tag{11}$$

$$\sin\beta = \frac{1}{\sqrt{1-\frac{v^2}{c^2}\cos^2\varphi'}} \times \frac{\sin\beta' + \frac{v}{c}\cos\varphi'}{\sqrt{\gamma^2\left[\sin\left(\beta'+\varphi'\right) + \frac{v}{c}\right]^2 + \cos^2\left(\beta'+\varphi'\right)}}, \tag{12}$$

where, $\gamma = 1/\sqrt{1-v^2/c^2}$ and primed parameters are in the moving frame. By these equations the angle of reflection from moving mirror can be obtained. In Ref. [3] the authors showed that in the special case where the mirror moves perpendicular to its surface, $\varphi = \varphi' = 90^\circ$ we have

$$\cos\beta = \frac{\left(1+\frac{v^2}{c^2}\right)\cos\alpha + 2\frac{v}{c}}{(1+\frac{v^2}{c^2}) + 2\frac{v}{c}\cos\alpha}. \tag{13}$$

Applying simple conversion $v \to v\sin\varphi$ in Eq. (13) we get to Eq. (1). These results show that, to consider the inclined mirror, we can impose simple coordinate rotation to get a vertical mirror with inclined velocity for simplification (Fig. 2). Therefore, an inclined mirror with the velocity v has equivalent behavior such as the vertical mirror with the velocity $v\sin\varphi$, since mirror movement in own direction with the velocity $v\cos\varphi$ does not have any effect on the reflection angle.

We have prepared a geometric structure in Fig. 3 to get the pulse length after reflection from the moving mirror in the direction of propagation. All calculations are done in the laboratory frame and we do not need Lorentz transformations. The pulse length before reflection is L_i. After pulse reflection at time t, when endpoint of the pulse reaches the surface of the mirror at point B, the first point of the pulse which at time t_0 had hit the mirror, reaches point A. We denote the straight line connecting points A and B by L_r and time difference $t - t_0$ by Δt. L_r is the distance between the first and last points of reflection. The actual length of the pulse after reflection is the image of L_r in the direction of propagation which we denote by $L_{r||}$ and is equal to $\overline{BD}$ the distance between points B and D. Since