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Mirror symmetry multi-wing attractors generated from a novel four-dimensional hyperchaotic system

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ABSTRACT

This research paper introduces mirror symmetry multi-wing attractors from a novel four-dimensional hyperchaotic system. A mirror symmetry conversion-based approach is developed and verified. To confirm the existence of hyperchaotic system, Lyapunov exponent spectrums are investigated. Furthermore, the improved module-based circuit designs of mirror symmetry multi-wing hyperchaotic attractors are further implemented. The proposed novel mirror symmetry multi-wing hyperchaotic attractors are very useful for deliberate generation of chaos in applications.

1. Introduction

Recently, chaos has been investigated and studied in recent years because of the great applications in secure communication, data encryption, flow dynamics as well as in engineering applications [1–5]. In 1963, E. N. Lorenz discovered the first double-wing butterfly attractor, which has become a paradigm in chaos [6,7]. The generalized Lorenz system family, such as Chen system, Lü systems, Liu system, have been discovered successively which can generate topologically non-equivalent various double-wing chaotic attractors [8–15]. These discoveries not only have stimulated the depth study of the nonlinear dynamic system theory, but also unfolded a promising prospect of their relevant applications in engineering technologies.

In the last decade, research for more complex chaotic systems has led to the finding of multi-wing chaotic attractors [16–18]. Compared with the double-wing and multi-wing chaotic systems, complex grid multi-wing hyperchaotic systems exhibit more complicated dynamical behaviors and better performance in many other technological application fields, such as secure communication systems, where the information is encrypt and the security performance can be improved by using these complex grid multi-wing hyperchaotic signals. One may ask whether or not there is a possible way further to construct various complex grid multi-wing hyperchaotic systems? This paper gives a positive answer to the question.

This paper investigates a new method for generating mirror symmetry grid multi-wing hyperchaotic system. Specifically, by introducing a mirror symmetry conversion-based approach in a hyperchaotic system, various mirror symmetry grid multi-wing hyperchaotic attractors can be simulated, which are also implemented by the improved module-based circuit method. To confirm the existence of hyperchaotic system, Lyapunov exponent spectrums are investigated. One characteristic of the proposed approaches lies in their generality, which is also suitable for constructing other grid multi-wing hyperchaotic systems.

2. A new double-wing hyperchaotic system

A four-dimensional double-wing hyperchaotic system is given as below:

$$\begin{cases} \mathrm{d}x/\mathrm{d}\tau = ax + by - yz \\ \mathrm{d}y/\mathrm{d}\tau = x - w \\ \mathrm{d}z/\mathrm{d}\tau = y^2 - cz \\ \mathrm{d}w/\mathrm{d}\tau = dy \end{cases} \tag{1}$$

where $a = -2$, $b = 6.7$, $c = 1$, $d = 0.5$. According to Eq. (1), the numerical simulation results of double-wing hyperchaotic attractors are depicted in Fig. 1.

It should be noted that, in our circuit design, the power supply for all operational amplifiers is ± 15V, and the saturation value for all operational amplifiers is $E_{sat} = \pm 13.5$V. From Fig. 1, the linear dynamic range of state variables x, y, z, w are in $[-15, 15]$, $[-8, 8]$,

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0030-4026/

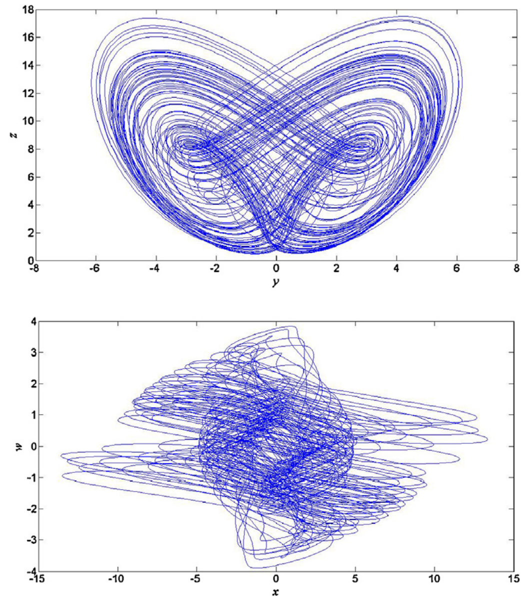

Fig. 1. Double-wing hyperchaotic attractors before variable-scale reduction.

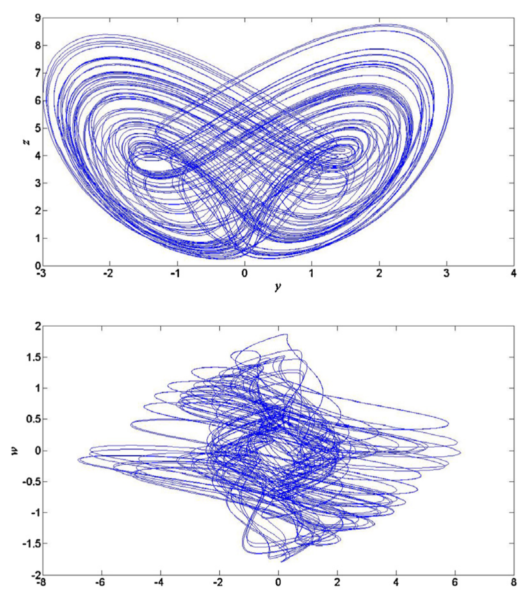

Fig. 2. Double-wing hyperchaotic attractors after variable-scale reduction.

[0, 18] and [−4, 4], respectively, which exceeds the linear dynamic range of all operational amplifiers. Therefore, the variable-scale reduction in Eq. (1) is required.

Let variable-scale reduction factor be E. After some simplified treatment, Eq. (1) is transferred as follows:

$$\begin{cases} dx/d\tau = ax + by - \left(1/E\right) yz \\ dy/d\tau = x - w \\ dz/d\tau = \left(1/E\right) y^2 - cz \\ dw/d\tau = dy \end{cases} \tag{2}$$

where $a = -2$, $b = 6.7$, $c = 1$, $d = 0.5$, and variable-scale reduction factor $E = 1/2$. Based on Eq. (2), the numerical simulation results of double-wing hyperchaotic attractors are shown in Fig. 2.

3. Generation of multi-wing hyperchaotic attractors via switching control

In this section, a switching controller can be introduced to extend the saddle-focus equilibrium points with index 2. From Eq. (2), by using the witching controller $S_C(y)$, one gets:

$$\begin{cases} dx/d\tau = ax + by - \left(1/E\right) yz \\ dy/d\tau = x - w \\ dz/d\tau = f\left(y\right) - cz \\ dw/d\tau = dy \end{cases} \tag{3}$$

where $a = -2$, $b = 6.7$, $c = 1$, $d = 0.5$, $E = 1/2$, the mathematical expression of $f(y)$ is:

$$f\left(y\right) = H_0 y^2 + S_C\left(y\right) = H_0 y^2 + \sum_{i=1}^{N} H_i \left[sgn\left(y + C_i\right) - sgn\left(y - C_i\right) - 2\right] \tag{4}$$

In Eq. (4), $N \geq 1$ is a positive integer, $i = 1, 2, 3, \ldots, N$. H_i and C_i are recursive parameters, obtained by:

$$\begin{cases} H_0 = \left(1/E\right) \\ H_i = AE/k_i \\ C_i = 0.5\left(n+1\right) AE \end{cases} \tag{5}$$

Let $N = 4$, $E = 1/2$, $A = 3.51$, $k_1 = 0.621$, $k_2 = 0.378$, $k_3 = 0.3$ and $k_4 = 0.273$. From Eq. (5), one gets $H_0 = 2$, $H_1 = 2.825$, $H_2 = 4.645$, $H_3 = 5.854$, $H_4 = 6.432$, $C_1 = 1.755$, $C_2 = 2.675$, $C_3 = 3.525$, $C_4 = 4.295$.

According to Eqs. (3) and (4), letting $N = 1, 2, 3, 4$, respectively, one can obtain the numerical simulation results of multi-wing hyperchaotic attractors depicted in Fig. 3.

4. Construction of mirror symmetry multi-wing hyperchaotic attractors

In this section, a mirror symmetry conversion-based approach is developed here to generate various mirror symmetry multi-wing hyperchaotic attractors.

Theorem 1. Assume that the general form of 4-D system family is

$$\begin{cases} dx/dt = f_1\left(x, y, z, w\right) \\ dy/dt = f_2\left(x, y, z, w\right) \\ dz/dt = f_3\left(x, y, z, w\right) \\ dw/dt = f_4\left(x, y, z, w\right) \end{cases} \tag{6}$$

If the corresponding conversion of (6) with respect to z-axis is given by

$$\begin{cases} dx/dt = f_1\left(x, y, |z| \pm z_0, w\right) \\ dy/dt = f_2\left(x, y, |z| \pm z_0, w\right) \\ dz/dt = sgn\left(z\right) \cdot f_3\left(x, y, |z| \pm z_0, w\right) \\ dw/dt = f_4\left(x, y, |z| \pm z_0, w\right) \end{cases} \tag{7}$$

Then the solution of (7) is mirror symmetry with respect to z-axis. Where z_0 is a coordinate translation constant, which makes the solution of (6) always satisfying $z \geq 0$ (or $z \leq 0$).