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# The location of limit cycles and the associated mechanism in the state switched nonlinear system

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ABSTRACT

By introducing switching law associated with the values of the state variables, a switched mathematical model is established. Poincaré map of the whole switched system is defined by suitable local sections and local maps, and the formal expression of its Jacobian matrix is obtained. The location of the fixed point corresponding to the limit cycle of the switched system is calculated by the shooting method. To investigate switching behavior of this system, the equilibrium points and their bifurcations of the subsystems are derived. An interesting switching behavior, i.e., the so-called 4T-focus/focus/focus periodic switching is explored in detail to present the mechanism of the movement. With the increase of the parameter, the turning points on the switching surface may be attracted by different attractors of the subsystem, causing the turning points decrease from four to two. Then the system forms other types of periodic solutions. Furthermore, period-decreasing and period-adding sequences have been obtained, which can be explained by the changes of the duration time in the subsystems.



## 1. Introduction

Switched dynamical systems are one of the most interesting class of piece-wise smooth systems that arise in scientific problems and engineering applications such as the electrical circuits [1], chemical processing [2], communication networks [3]. Typically, a switched dynamical system is composed of a family of subsystems and rules that governs the switching of them. Generally, switching rules are determined by the values of the state variables or related to the fixed time for the occurrence of the alteration [4,5]. When the switching rules are satisfied, the vector field may alternate from one dynamical subsystem to another, leading to the vector field is not differentiable at turning point [6–8]. Because of the wide existence of switches, the behaviors of switched system have received much interest during the last decades and a lot of results have been reported [9–12]. In [9], some new exponents were defined by which the essential patterns that guarantee the stability of fast switching systems could be figured out and the capacity and efficiency of random switch for stabilizing a switched system were described. In [10], new type of characteristic exponent was introduced to capture the stability feature of continues-time switched systems and two criteria of asymptotic exponential stability for linear and nonlinear case were obtained. In [11], the stabilization of switched nonlinear systems with passive and non-passive subsystems was studied via the average dwell time method. In [12], some sufficient conditions were obtained to ensure global asymptotical stability and global robust stability of the unique equilibrium of switched neural networks.

Up to now, much attention has been paid to the stability, controllability, reachability, observability and design of the switched system [13–17]. However, little work has been done in the dynamical evolution with the variation of the parameters, the difference between the behaviors caused by two types of switches, respectively, and the parameter bifurcations associated with the switches as well as the mechanism of the complexity.

Here we consider a switched system alternating between two subsystems described by Rössler system and Lorenz oscillation with switches defined on the conditions related to the state variables. Some interesting phenomena such as periodic switching, period-decreasing, period-adding sequences have been obtained. Based on the equilibrium points analysis of the two subsystems as well as the critical behaviors at the switches, the mechanism related to the periodic orbits observed are presented to account for the evolutions of the trajectories.

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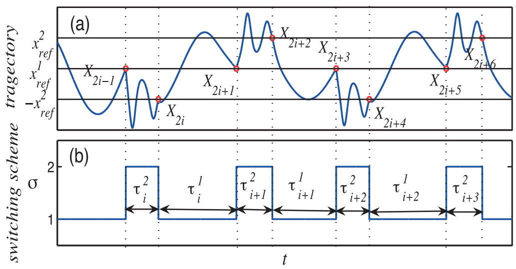


**Fig. 1.** (a) Trajectory partition and (b) the switching scheme.

## 2. The model of switched system

Let us consider a state switched system, which alternates between Rössler system and Lorenz oscillator, written in the form

$$\frac{d\mathbb{X}}{dt} = f_\sigma(\mathbb{X}), \quad \sigma \in \{1,2\}, \tag{2.1}$$

where $\mathbb{X} = (x, y, z)^T . f_\sigma, \sigma \in \{1, 2\}$ are the vector fields with $f_1(\mathbb{X}) = (-y - x, x + \mu y, xz - \sigma z + \eta)^T$ and $f_2(\mathbb{X}) = \left(\alpha(y - x), x(\delta - z) - y, xy - \beta z\right)^T$. $\sigma$ is a switching scheme. When $\sigma = 1$ means the subsystem 1, named SW1, be active, while $\sigma = 2$ means the subsystem 2, named SW2, be active.

In order to investigate the typical dynamics of the switched system, the switch scheme $\sigma$ related to three switching boundaries $S_1 = \{\mathbb{X} \in \mathbb{R}^3 \mid x = x^1_{ref}\}$ and $S_{2,3} = \{\mathbb{X} \in \mathbb{R}^3 \mid x = \pm x^2_{ref}\}$ are introduced, shown in Fig. 1.

For the trajectory of the switched system starting from an initial point $\mathbb{X}_0$, governed by SW1, i.e., $\sigma = 1$. Once one of the state variable $x$ passes across the reference value $x^1_{ref}$ twice at $t = \tau^1_0$, it may then be governed by SW2, i.e., $\sigma$ change from 1 to 2. The trajectory moving according to SW2 may change back to the vector field $f_1(\mathbb{X})$ when the trajectory passes across the reference value $x^2_{ref}$ or $-x^2_{ref}$ four times at $t = \tau^1_0 + \tau^2_1$, until the state variable $x$ passes across $x^1_{ref}$ twice again and the motion then continues as above. Based on the switching scheme described above, two sequences $(\mathbb{X}_i, i \in N)$ and $(\tau^k_i, i \in N, k \in \{1, 2\})$ can be obtained,

$$\mathbb{X}_1 \xrightarrow{\tau^2_1} \mathbb{X}_2 \xrightarrow{\tau^1_1} \mathbb{X}_3 \xrightarrow{\tau^2_2} \mathbb{X}_4 \xrightarrow{\tau^1_2} \mathbb{X}_5 \xrightarrow{\tau^2_3} \mathbb{X}_6 \xrightarrow{\tau^1_3} \mathbb{X}_7 \xrightarrow{\tau^2_4} \cdots$$

where the points $(\mathbb{X}_i, i \in N)$ are the turning points and $(\tau^1_i, i \in N)$ and $(\tau^2_i, i \in N)$ are the duration time in the two subsystems, respectively. Therefore, the trajectory of the whole switched system can be divided into two parts, one is governed by SW1,expressed as

$$\mathbb{X}(t) = \Phi(t, \mathbb{X}_{2i}), \quad t \in \bigcup_{i=0}^{\infty} \left[ \sum_{j=0}^{i} (\tau^1_j + \tau^2_j), \sum_{j=0}^{i} (\tau^1_{j+1} + \tau^2_j) \right], \tag{2.2}$$

the other is determined by SW2, i.e.,

$$\mathbb{X}(t) = \Psi(t, \mathbb{X}_{2i+1}), \quad t \in \bigcup_{i=0}^{\infty} \left[ \sum_{j=0}^{i} (\tau^1_{j+1} + \tau^2_j), \sum_{j=0}^{i} (\tau^1_{j+1} + \tau^2_{j+1}) \right], \tag{2.3}$$

where $\tau^2_0 = 0$. According to the switching scheme, it is easy to know that the points $(\mathbb{X}_{2i-1}, i = 1, 2, \cdots)$ are on the switching surface $S_1$, while the points $(\mathbb{X}_{2i}, i = 1, 2, \cdots)$ are on the switching surface $S_{2,3}$.

## 3. The location of limit cycles

In this section, we investigate the generation of the period oscillation of the switched system (2.1). The multiple-shooting method present in [18] will be adopted to locate limit cycle.

With the definition of the trajectory of the switched system given by (2.2) and (2.3), the following two local maps are defined

$$\begin{aligned} T_1 : S_{2,3} &\longrightarrow S_1 \\ \mathbb{X}_{2i-1} &\longmapsto \mathbb{X}_{2i} = \Psi(\tau^2_i, \mathbb{X}_{2i-1}), \\ T_2 : S_1 &\longrightarrow S_{2,3} \\ \mathbb{X}_{2i} &\longmapsto \mathbb{X}_{2i+1} = \Phi(\tau^1_i, \mathbb{X}_{2i}). \end{aligned} \tag{3.1}$$

where $i = 1, 2, \ldots$. Assume that the switching surface $S_{2,3}$ is the poincaré section, the poincaré mapping $T$ from $S_{2,3}$ to $S_{2,3}$ can be expressed as

$$T(\mathbb{X}_{2i-1}) = T_2 \circ T_1(\mathbb{X}_{2i-1}) = \Phi(\tau^1_i, \Psi(\tau^2_i, \mathbb{X}_{2i-1})). \tag{3.2}$$

Essentially, the periodic oscillation of the switched system (2.1) is equivalent to the existence of the fixed point of the poincaré mapping $T$, namely

$$\mathbb{X}^* - T(\mathbb{X}^*) = 0. \tag{3.3}$$

Since the analytic expression of the poincaré map $T$ is unknown, and in order to compute the fixed point, we need to compute its Jacobian matrix

$$DT = DT_2 \times DT_1 = \frac{\partial \mathbb{X}_{2i+1}}{\partial \mathbb{X}_{2i}} \times \frac{\partial \mathbb{X}_{2i}}{\partial \mathbb{X}_{2i-1}} \tag{3.4}$$

Notice that $\mathbb{X}_{2i} = \Psi(\tau^2_i, \mathbb{X}_{2i-1})$, $\mathbb{X}_{2i+1} = \Phi(\tau^1_i, \mathbb{X}_{2i})$ and the duration time $\tau^1_i$ and $\tau^2_i$ are dependent on the state variation $\mathbb{X}_{2i}$ and $\mathbb{X}_{2i-1}$, respectively, then the Jacobian matrix $\frac{\partial \mathbb{X}_{2i}}{\partial \mathbb{X}_{2i-1}}$ and $\frac{\partial \mathbb{X}_{2i+1}}{\partial \mathbb{X}_{2i}}$ can be written as the follow form

$$\frac{\partial \mathbb{X}_{2i}}{\partial \mathbb{X}_{2i-1}} = \frac{\partial \Psi}{\partial \tau^2_i} \times \frac{\partial \tau^2_i}{\partial \mathbb{X}_{2i-1}} + \frac{\partial \Psi}{\partial \mathbb{X}_{2i-1}} \tag{3.5}$$

$$\frac{\partial \mathbb{X}_{2i+1}}{\partial \mathbb{X}_{2i}} = \frac{\partial \Phi}{\partial \tau^1_i} \times \frac{\partial \tau^1_i}{\partial \mathbb{X}_{2i}} + \frac{\partial \Phi}{\partial \mathbb{X}_{2i}} \tag{3.6}$$

where $\frac{\partial \Psi}{\partial \tau^2_i} = f_2(\mathbb{X}_{2i-1})$, $\frac{\partial \Phi}{\partial \tau^1_i} = f_1(\mathbb{X}_{2i})$, while the elements of the matrices $\frac{\partial \Psi}{\partial \mathbb{X}_{2i-1}}$ and $\frac{\partial \Phi}{\partial \mathbb{X}_{2i}}$ can be computed by the following variational equations

$$\begin{cases} \dfrac{d}{dt}(D\Psi) = f_{2\mathbb{X}} \times D\Psi \\ \left.\dfrac{\partial \Psi}{\partial \mathbb{X}_{2i-1}}\right|_{t=0} = I \end{cases} \tag{3.7}$$

$$\begin{cases} \dfrac{d}{dt}(D\Phi) = f_{1\mathbb{X}} \times D\Phi \\ \left.\dfrac{\partial \Phi}{\partial \mathbb{X}_{2i}}\right|_{t=\tau^2_i} = I \end{cases} \tag{3.8}$$

from $t = 0$ to $t = \tau^2_i$ and $t = \tau^2_i$ to $t = \tau^2_i + \tau^1_i$, respectively. Where $f_{1\mathbb{X}}$ and $f_{2\mathbb{X}}$ are the Jacobian matrix of $f_1$ and $f_2$. $I$ is an identity matrix. Notice that $\mathbb{X}_{2i}$ and $\mathbb{X}_{2i+1}$ are on the switching surfaces $S_1$ and $S_{2,3}$,respectively. That is to say

$$q_1(\mathbb{X}_{2i}) \equiv x - x^1_{ref} = 0 \tag{3.9}$$

$$q_2(\mathbb{X}_{2i+1}) \equiv x \pm x^2_{ref} = 0 \tag{3.10}$$